On Old Babylonian Mathematical Terminology and its Transformations in the Mathematics of Later Periods*

JENS HØYRUP

ABSTRACT

Third-millennium (BCE) Mesopotamian mathematics seems to have possessed a very restricted technical terminology. However, with the sudden flourishing of supra-utilitarian mathematics during the Old Babylonian period, in particular its second half (1800–1600 BCE) a rich terminology unfolds. This mostly concerns terms for operations and for definition of a problem format, but names for mathematical objects, for tools, and for methods or tricks can also be identified. In particular the terms for operations and the way to structure problems turn out to allow distinction between single localities or even schools. After the end of the Old Babylonian period, the richness of the terminology is strongly reduced, as is the number of known mathematical texts, but it presents us with survival as well as innovations.

Apart from analyzing the terminology synchronically and diachronically, the article looks at two long-lived non-linguistic mathematical practices that can be identified through the varying ways they are spoken about: the use of some kind of calculating board, and a way to construct the perimeter of a circle without calculating it – the former at least in use from the 26th to the 5th century BCE, the later from no later than Old Babylonian times and surviving until the European 15th century CE.

Keywords: Terminology, Mathematical, Old Babylonian mathematics, continuity, Mesopotamian mathematics.

* First presented as a contribution to Seminar SAW "History of Mathematics, History of Economical and Financial Practices", session of 15 June 2012: Names of operations : Meaning of the terms and sociolinguistic analysis". The article mainly synthesizes earlier work of mine under this particular perspective.The "Old Babylonian" period lasted from *ca* 2000 to *ca* 1600 BCE ("middle chronology"). With very few possible exceptions, the mathematical texts we know were produced during its second half.

Gaṇita Bhāratī Vol. 40, No. 1 (2018) pages 53-99
Corresponding Author E-mail: jensh@ruc.dk
DOI : https://doi.org/10.32381/GB.2018.40.01.3

"... wo Begriffe fehlen, da stellt ein Wort zur rechten Zeit sich ein" (where concepts fail, there at the right moment a word finds its place), thus Mephisto in Goethe's *Faust* (I, 1995*f*). This may be true in (pseudo-)sciences like theology (of which Mephisto speaks) and philosophy, and according to certain philosophies of mathematics (those which consider only the formal game of symbols signifying nothing beyond their appearance within axioms) about this "queen and handmaid of science". However, exactly this epithet raises Eugene Wigner's famous question [1960] about "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" – in order to be effective (not to speak of *unreasonably* effective), the words/terms of mathematics need correspond to concepts, not only understood as networks of operations within a space defined by outspoken or tacitly assumed axioms but also as networks that reach beyond the space of abstract beer-mugs, chairs and tables attributed to Hilbert. When they do not, we get instead "unreasonable *in*effectiveness", as K. Vela Velupillai [2005: 849] states about mathematics in economics: "Unreasonable, because the mathematical assumptions are economically unwarranted; ineffective because the mathematical formalisations imply non constructive and uncomputable structures".

Certainly, when it comes to Mesopotamian mathematics we know the concepts and the operations almost exclusively through the words of texts – the exceptions being some geometrical drawings; some weights and measuring sticks corresponding to metrological units; some tables of technical constants that must be understood within the limits of the physically and physiologically possible or in agreement with artefacts (bricks etc.) that have been excavated; in Late Babylonian times (in mathematical astronomy) in agreement with celestial phenomena which we know in other ways; and a bit more. Our own knowledge about the structure of elementary arithmetic and elementary Euclidean geometry may also help us (tables of reciprocals stating that the igi[1] of 7 does not exist or simply omitting this line correspond well to our idea that 7 does not divide any power of 60), but should of course be used with care.

Nonetheless, Mephisto and the clash between Wigner and the folklore Hilbert should warn us that remaining within the walled magic garden of words may delude.

[1] Spaced writing renders logograms, mostly for Sumerian words. Akkadian is transcribed in *italics*. Unexplained sign names appear as SMALL CAPS.

Long-living Practices

So, let us start with two long-living *practices*, reflected in words that change. The first has to do with the determination of the circumference from the diameter of a circle. In Mesopotamia, the ratio between the two magnitudes was supposed to be 3:1.[2] The basic circle perimeter was the circumference, but in cases where the perimeter has to be found from the diameter[3], the operation is not a "raising" (*našûm*/íl) as one would expect from this being the operation invariably used in multiplication with technical constants (see below, after note 47) but a *tripling* (*šullušum*) – or the diameter is "repeated in three steps".

This could be an unimportant though unexplainable quirk, but Greek practical geometry as contained in the pseudo-Heronian *Geometrica*[4] and as quoted by Heron in the *Metrica*[5] shows that it is not. On all occasions, the terms τρισσάκις and τριπλάσιον are used even when neighbouring multiplications are ἐπὶ *n*; afterwards, a supplementary Archimedean seventh is added.[6] Even *if* we believe that the Greek

[2] Old Babylonian scribes were probably aware that this was a practical value or approximation, since they also knew 3:1 to be the ratio between the perimeter and the diameter of a regular hexagon [TMS, 24]. Alternatively, the value has recently been proposed [Brunke 2011: 113] "to be the result of a specific Babylonian way to *define* the area measure of a circle". Since nothing suggests the Babylonians to have bothered about mathematical definitions, this idea can probably be discarded. Whether the possible alternative ratio $3^1/_8$:1 suggested by the text YBC 8600 [MCT, 57–59] was supposed to be a better approximation or was just adopted (*if* it was really meant) for ease of calculation (as supposed by Otto Neugebauer and Abraham Sachs) is hardly decidable. The suggestion of E. M. Bruins to find the same approximation in an igi.gub table (a table of technical constants) from Susa [TMS, 26, 28] can be discarded, since šár means neither "circle" nor "more perfect circle".

[3] BM 85194, obv. I 47–48; Haddad 104, I 4, 14, 26, 40, II 7, 26, 36, 42, III 14, 20, 26.

[4] These treatises were published by Heiberg [1912] as one, even though he clearly saw and expressed [1914: xxi] that at least two independent treatises are involved; the structure of the *Opera omnia* may already have been determined when Heiberg was called into the project at Wilhelm Schmidt's death, even though this is not said clearly in the beginning of the introduction [Heiberg 1912: iii*f*]. In any case, the bulk of the conglomerate comes from two treatises (even they composite) represented most fully by Mss AC and MS S. The relevant passages are Mss AC, 17.10, 17.29; SV: 17.8 S:22.16; and S:24.45 (S.24 is actually a third small but still composite treatise). See [Høyrup 1997].

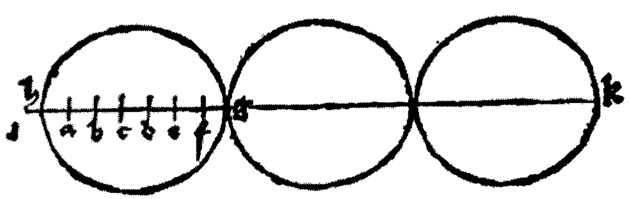
[5] I.xxx, xxxi, ed. [Schöne 1903: $74^{5,25}$].

[6] Except in the *Metrica*, where Heron distinguishes "the ancients" who took the perimeter to be the triple of the diameter, and the recent workers who take it to be the triple, and one seventh added.

practitioners had translated the verbal rules of Mesopotamian forerunners, this level of philological precision would be astonishing.

The explanation is found in two vernacular texts from the late Middle Ages, one in Middle High German and one in Old Icelandic. The former is Mathes Roriczer's *Geometria deutsch* from c. 1486 or shortly afterwards. This is how it tells how to make a round line straight [Roriczer 1497]:[7]

> Nernach so einer ein gerunden riß scheitrecht machen wil dz d scheitgerecht riß und dz gerund ein leng sey so mach drey gerunde neben ein ander und tayl dz erst rund in siben gleiche teil mit den puchstaben verzeichnet h.a.b.c.d.e.f.g: Darnach alsz weit vom .h. in das .a. ist da setz hindersich ein punckt da setz ein .i. Darnach alß weit von dem .i. piß zu dem .k. ist Gleich so lang ist der runden riß einer in seiner rundung der drey neben ein and sten des ein figur hernach gemacht stet.



In literal translation:

> When somebody wishes to make a round line straight, so that the straight line and the round are one length. Then make three rounds next to one another, and divide the first round into seven equal parts, designated with the letters *h a b c d e f g*. Then as far as it is from *h* to *a*, set behind it a point, and set an *i*. Then as far as it is from the *i* to the *k*, so long is one of the rounds in its rounding of the three that stand next to each other, of which a figure stands made hereafter.

The old Icelandic manuscript A.M. 415 4to from the early 14th century, on its part, states (fol. 9^{v}) that "the measure around the circle is three times as long as its width, and a seventh of the fourth width",[8] obviously a reference to a similar construction.

[7] According to [Shelby 1977: 120*f*], this differs from Roriczer's original only in orthography.

[8] "Ummæling hrings hvers primr lutum lengri en bréidd hans ok sjaundungr of enni fiorðo breidd" [ed. Beckman & Kålund 1914: 231*f*].

As we see, these two medieval texts tell how to *construct* the length of the perimeter, not how to calculate it.[9] This construction must have been used by master builders at least from Old Babylonian times until late medieval Gothics, with only a marginal change taking into account Archimedes's improved approximations. A practical construction, not philological precision, explains the accuracy of the "translation" of the rule.

The other example remains within the Mesopotamian orbit. As shown by Christine Proust [2000], Mesopotamian calculators made use of a reckoning board called "the hand", from the accountants of the 26th century[10] until the Seleucid mathematical astronomers. The name (šu/*qātum*) is likely to have been transmitted at the level of words (unless we imagine that a real hand can have been used to carry five levels for ones and five levels for tens and permit easy transfer of "calculi" between these ten handheld cases).

However, a strange continuity at the level of semantics seems to be better explained at the level of operations. As we shall see, Old Babylonian texts use ĝár.ĝár and zi logographically for *kamārum* and *nasāḫum*, respectively, that is, for "heaping" addition and for subtraction by removal (cf. below). However, a well-known passage from "Šulgi-Hymn B", l.17 [ed. Castellino 1972: 32] claims that this 21st-century king has learned zi.zi ĝá.ĝá šid nìg.šid, "to subtract and add, counting and accounting".[11] zi.zi and á.ĝá are *marû*-stems of zig, "to rise", and ĝar, "to place", respectively [Thomsen 1984: 305, 322], and probably mean "to

[9] The related 15th-century manuscript AM. 624 [ed. Beckman & Kålund 1914: 99] instead ascribes to "geometrici" the *calculational* rule that "sircumferenncia hvers hrings hafi i ser pren diametur sin ok hin 7. hlut af diametril", "the circumference of every circle contains thrice its diameter and the 7th share of the diameter" – quite similar to the rule of the Greek practitioners. Similar formulations are absent from Latin agrimensor or other "sub-Euclidean" geometry as we know it through [Bubnov 1899], [Blume et al 1848] and [Thulin 1913], but Latin learning might have known it through Macrobius's *Commentary on the Dream of Scipio* [ed. Eyssenhardt 1868: 555*f*]. Though the terminologies are different (Macrobius speaks of *orbis* and not of circumference), a borrowing via the written tradition is thus more likely than yet another translation of the construction into a rule for computation.

[10] All Mesopotamian dates are evidently BCE. For convenience, I follow the Middle Chronology where this distinction is pertinent.

[11] Thus the translation in [Sjöberg 1976: 173]; Castellino misses the mathematical point.

take up" and "to put down" – namely on the reckoning board. These are not the meanings of *nasāḫum* and *kamārum*, and it appears that the Sumerograms have been selected for semantic proximity, not identity (as happened in other cases, too), and even abbreviated (into zi) or changed (into ĝár.ĝár).

In a small batch of mathematical texts produced in the environment of scholar-scribes in the fifth century (see below, before note 59), subtraction is spoken of as nim, which in Old Babylonian texts occurs occasionally as a logogram for the "raising" multiplication (belonging to the same semantic cluster as *našûm* and íl, see below, after note 46). In the fifth century, the meaning seems to be "to take up" or "lift", that is, to refer once again to the reckoning board.

In the Seleucid text BM 34568, we similarly find for instance "16 ta 25 nim *ma ri -ḫi* 9", "16 from 25 you lift: remains 9". Not knowing the fifth-century intermediate step, Otto Neugebauer took ta to be a genuine Sumerian suffix and translated "von 16 bis 25 steigst du auf, und es bleibt 9". Instead, the fifth-century text shows us that ta is nothing but a logogram for *ina*, "from", the underlying phrase as a whole being Akkadian – with a reference to an operation on the reckoning board.

In consequence, the shift from one Sumerian term to another one must be explained not at the level of textual transmission or translation but as two instances of putting the same material operation into Sumerian words.

The levels of terminology

After this warning that words – and in particular written words – are not the only instruments for, and not the only transmitters of knowledge, let us nonetheless turn to written words – first, and mainly, those used in Old Babylonian mathematics.

Such words belong at many levels. Restricting myself to what I am going to discuss, I shall list the following categories:

- First, there are *names for tools*. The "hand" was already mentioned, but tables are also tools. To the extent they can be shown to possess a name, they are clearly understood as such, not just as a list of analogous items.
- Then there are *names for methods and tricks*. A delimitation of the range of variations covered by a particular name may be an important means for characterizing the type of mathematical thought within which they serve.
- Third, there are *terms and phrases used to structure* a mathematical text – for instance, to indicate that it constitutes a problem, and to delimit the various steps in the presentation and solution of a problem.
- Fourth, there are *names for mathematical objects*, also informative in different ways, not least when they conflate what for us seem to be different objects.
- Fifth and finally, there are *terms for mathematical operations*.

Names for tools

Old Babylonian mathematics made ample use of the tables connected to place-value computation; some uses – first of all of the multiplication table – are only implicit.[12] But occasionally the texts refer explicitly to igi.gub constants, and the reciprocals they "detach" (*paṭārum*/du_8)[13] almost invariably appear in the standard table of reciprocals. In Old Babylonian problems about *igûm* and *igibûm*, "the reciprocal" and "its reciprocal", these are also pairs that appear in the standard table (5 and 12, 1 30 and 40, 1 4 and 56 15, 1 40 and 36, 1 20 and 45, 1 12 and 50, 2 and 30, 1 40 and 36).[14]

However, these are nothing but references to items from the tables, and not to the tables as entities as such. Nor do the tables themselves carry titles. However, one problem text carries an explicit reference to a table.

This is the text BM 85200 + VAT 6599, famous for sometimes treating irreducible cubic problems about a parallelepipedal "excavation" (alongside a number of problems of the first and second degree about the same configuration –

[12] The term a.rá is used repeatedly in AO 8862, but most of the multiplications spoken of thus are not found in the tables. We may conclude that it just stands for the multiplication of a number with a number, as it *also* does in the multiplication table. The phrase *A*.e *s* íb.si_8, used in many tables of inverse squares [MKT I, 70*f*], similarly appears in many problem texts, but again often in cases that are not listed in the tables. It thus cannot be taken as a reference to the table but only as a phrase shared with these.

[13] Here and everywhere in the following I make use of the "standard translations" used in the "conformal translations" of [Høyrup 2002a].

[14] [MCT, 129*f*], [MKT I, 197, 346–349], [Friberg 2007: 252–254].

see [Høyrup 2002a: 137–162]). In rev. I 23, where 4`12[15] is to be factorized as $p{\cdot}p{\cdot}(p{+}1)$, we find "*i-na* íb. si$_8$ 1 daḫ.ḫa 6 [$^{\text{!}}$erasure$^{?}$] íb.s[i$_8$]". In this construction, the first íb. si$_8$ cannot be a verb, as everywhere else in the text, and daḫ ("to append") can never go with the preposition *ina*, "from". The only grammatically coherent interpretation is that *ina* governs the whole phrase "íb. si$_8$ 1 daḫ.ḫa", which must then mean something like "equalside, 1 appended". The whole phrase thus means "from 'qualside, 1 appended', 6 is equal". Tabulations of $p{\cdot}p{\cdot}(p{+}1)$, which would correspond perfectly to the name "equalside, 1 appended", have indeed been found – see [MKT I, 76*f*] and [Friberg 2007: 56–58].[16]

Beyond that, some "edubba texts"[17] refer to familiarity (or faulty familiarity) with the multiplication table; it is identified simply as a.rá, that is, by means of the operation term appearing explicitly or implicitly in each line – see [Friberg 2000: 152].

Since these table types carried a name, others probably also did. But these have not made it into the written texts (at least not those that have been read and interpreted).

A term connected to tables is *nadānum*/sum, "to give". The short text YBC 6295 tells how to proceed when "it does not give to you" (*la id-di-nu-kum*) the cubic side of a number – see [Høyrup 2002a: 65]. In general, the term is mostly used for the outcome of calculations in the place-value system (one text groups applies it more generally, see below); an origin in Ur III calculation (21st century) is not implausible.

We might expect "giving" to be coupled to "taking", and while the side of a square is normally stated as "what is equal" (íb. si$_8$ functioning as a verb) or "what

[15] My transcription of sexagesimal place value numbers follows the notation introduced by Assyriologists from 1911 onward. `, ``, ```,, ... stand for descending sexagesimal orders of magnitude (5´ thus means $5{\cdot}60^{-1}$),`, ``, ```, ... for ascending order of magnitude (5` thus means $5{\cdot}60$); when needed, ° is used to indicate "order zero". No similar indication is normally found in the texts (occasionally it is indicated in which part of the metrological table a number belongs). At times the ascription of an absolute order of magnitude is arbitrary, at times non-homogeneous calculations determine for us (if 15 is the square of 30, 30 must stand for $30{\cdot}60^{n}$, *n* being odd – and then most likely meant to be –1).

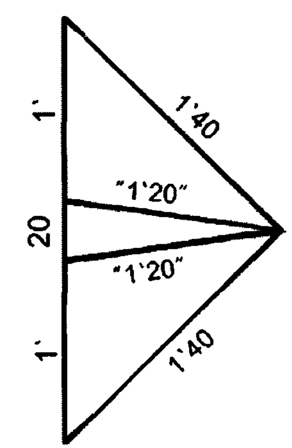
[16] VAT 8521 has a parallel reference to ba.si.1.lá, "equalside, 1 diminished" ([MKT I, 352], cf. [Friberg 2007: 1]). Whether the interest asked for is meant to be listed within a table *n.n.*(*n*–1) carrying this name or just to conform to this expression is unclear, however.

[17] That is, texts serving to inculcate scribal ideology and professional pride in scribe school students.

is the equalside" (íb. si$_8$ functioning as a noun), a few texts do "take" (*leqûm*) the equalside – thus Db$_2$-146, YBC 4675 and YBC 4662–4663. However, whether this is really meant as "taking" from a table is very doubtful. A number of texts "take" a fraction *of something* (whether determined as igi *n* or with an ordinal; yet *a reciprocal*, as occurring in the table, appears never to be "taken" but to be invariably "detached" [*paṭārum*/du$_8$]). Particularly striking is TMS XXV, which in rev. 6 and 9 "takes" the third (*šaluštum*) of 30 (which would not appear in any table), but "detaches" igi 40 and igi 30 in obv. 3, 5 and 9. So, even though values are *given* by tables, they seem not to be *taken* from them.

Names for methods and tricks

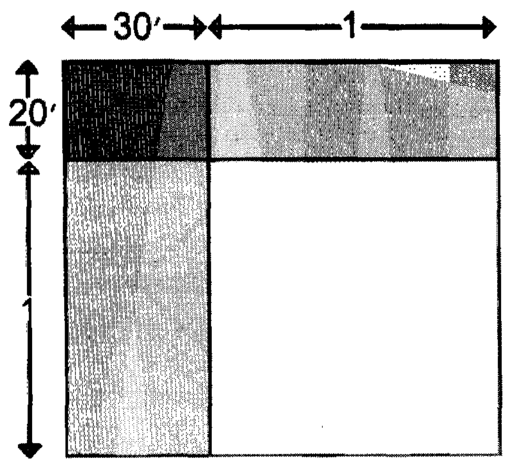
Two methods are mentioned by name in the problem texts. One is the *makṣarum*, derived from *kaṣārum*, "to bind together"; it may thus be translated "bundling". It occurs in three texts. The first is YBC 6295, just mentioned, which explains what to do when the cubic side of 3°22´30´´ is not "given". The method is to subdivide this volume into volumes 7´30´´, of which there turn out to be 8. We may see the subdivided cube as a "bundle" of 2×2×2 smaller cubes, and the initial line of the text states indeed that what follows is the "bundling of a (cubic) equilateral".



The second text mentioning the method is YBC 8633 [Høyrup 2002a: 254]. Here, a triangle with width 1` and longest length 1`40 is supposed to be subdivided into smaller triangles with sides 3, 4 and 5; since the original triangle (one of the outer wings of the adjacent diagram, here drawn in revelatory true proportions) is far from being right, this is not possible, but that is immaterial for the present discussion. The requested factor 20 is spoken of as "the bundling"; but in a heuristic summary the whole procedure is also spoken of as the "bundling of a trapezium (sag̃.ki.gud) with cross-over (*ṣiliptum*, i.e., diagonal)".

The third occurrence of the term is in the Susa text TMS XVII. The text is damaged, but here it appears to have to do with the partition of an area (the square on the sum of the sides of a rectangle) into sub-areas.

The other procedure spoken of by name turns up in the Susa text TMS IX, section 2 [Høyrup 2002a: 90–93]. This didactical text explains how to transform the sum of *the area*, *the length* and *the width* of a rectangle ($\sqsubset\!\sqsupset(l,w)+l+w$, $l = 30'$, $w = 20'$) into a rectangular area "by the Akkadian (method)", *i-na ak-ka-di-i*. At first, l is replaced by the rectangle $\sqsubset\!\sqsupset(l,1)$ and w by $\sqsubset\!\sqsupset(w,1)$. This generates a quasi-gnomon, a rectangle from which a square$\square(1)$ is lacking in a corner (see the diagram). "Appending" this square we obtain a rectangle $\sqsubset\!\sqsupset(l+1,w+1)$. After verifying that this rectangle fulfils the conditions, the explanation closes with the words "thus the Akkadian (method)", *ki-a-am-ak-ka-du-ú*.



Section 1 of the same text explains the trick of transforming $\sqsubset\!\sqsupset(l,w)+l$ into $\sqsubset\!\sqsupset(l,w+1)$. This trick receives no name. What is new in section 2 is thus the quadratic completion, albeit an idiosyncratic variant – actually not found anywhere else in the corpus, even though texts exist where it *could* easily have served (e.g., thrice in AO 8862). That a name should be reserved for a method that occurs in a single text

only (furthermore of late Old Babylonian date) is unlikely. It seems reasonable to assume that it refers to the method of quadratic completion in general, the normal type as well as whatever variants might turn up.

The *makṣarum*, we saw, also designated not a single procedure but a spectrum of (not too closely) related methods. According to the philological principle "Once is never, twice is always"[18] we may guess that this flexibility (or, if preferred, fuzziness) characterized the general view of Old Babylonian calculators of their panoply of methods.

Structuring terms and phrases

Restricting ourselves to mathematical texts proper (that is, omitting accounting and similar *uses* of mathematics), the corpus can be divided into three text types: tables; tablets for rough work; and problem texts – in didactical order, cf. [Proust 2008], tables being trained and learned by heart before being applied in elementary calculations, and problem texts being apparently a matter for specialists, outside the normal full curriculum (as we know it not least from Nippur) but presupposing it.

Tables were structured spatially, but apart from the words appearing in the single lines (a.rá, igi gál, etc.) not by means of words. Tablets for rough work are less uniform. Very often they contain numbers only – many examples are in [Robson 1999: 247–277]. But they *may* carry numbers as well as a geometric diagram – the most famous example being YBC 7289, which determines the diagonal of a square by means of an igi.gub value.[19] Finally, they may border the category of problem texts, and contain a question marked .bi en.nam ("its ... what?") and a possessive suffix .bi ("its") glued to the answer, as in the Nippur texts UM 29-15-192 and Ni 18, as well as CBS 11318.

en.nam is an innovation (already found in the few late 19th or early 18th-century problem texts from Ur, see [Friberg 2000: 139–144] and below), but use of .bi to mark a question or the quantity that is found goes back to Early Dynastic and Sargonic school texts (26th to 23d centuries BCE) [Powell 1976: *passim*; Foster & Robson

[18] My thanks to Eckhard Keßler for this jibe, which may go back to Ulrich von Wilamowitz-Moellendorf [Kahn 2003: 350].

[19] Three more – YBC 7290, YBC 11126 and YBC 7302 – are published in [MCT, 44], and nine in [Friberg 2007: 189–204].

2004, *passim*]. Direct continuity is not to be expected, however: the Sargonic texts regularly use the verb pàd (= pà), "to see", or the allograph pa for results found or to be found; this is totally absent from the Old Babylonian problem-close tablets for rough work (but not from all genuine problem texts, as we shall see below).

Problem texts: the text groups

Before we proceed with the discussion of the structuring of problems, a presentation of the groups into which the problem texts fall will be adequate.

A division of the Old Babylonian corpus into a "southern" and a "northern" group was first proposed by Neugebauer [1932: 6*f*]. It was elaborated by Albrecht Goetze [1945], who based his analysis mainly on orthography but also to some extent also on vocabulary (not terminology, since he did not take differences of meaning into account). Goetze divided the corpus of problem texts as known by then into six groups.

At a time when Assyriologists tended to regard texts containing too many numbers, in particular too many sexagesimal place-value numbers, as a "matter for Neugebauer" (who wrote his last paper on Babylonian *mathematics* together with Abraham Sachs in 1951 – mislaid but eventually published as [Neugebauer & Sachs 1984]), and during which most historians of mathematics still thought in terms of perennial "Babylonian mathematics", Goetze's analysis had little impact.

In 1996, having been invited by Hans Neumann to contribute to the Oelsner-Festschrift with the page limit "schreib so viel du willst!" ("write as much as you wish"), I took up the matter where Goetze had left it, including now the text groups from Ešnunna and Susa, which had not been known in 1945, and looking more specifically at terminology and structuring phrases (the paper was published as [Høyrup 2000]). With minor exceptions my analysis confirmed Goetze's division and Neugebauer's original hunch while adding the two new text groups. After the appearance of Jöran Friberg's study of the texts from early Old Babylonian Ur I included a revised version as chapter IX of [Høyrup 2002a], on which I draw heavily and mostly without specific references in the following.

According to this new analysis, the corpus of Old Babylonian problem texts falls into the following groups (I use Goetze's numeration as extended in [Høyrup 2000] and [Friberg 2000]):

1. According to Goetze "certainly to be localized in the South, in all probability Larsa".

2. According to Goetze "likewise a southern group". The important theme text BM 13901 has to be eliminated from the group; what remains may be designated "2A".

ii. The single tablet BM 13901, which Goetze had placed in "Group 2" for reasons which he himself characterized as circular, and which can now be seen to be irrelevant – but the text is certainly also southern.

3. According to Goetze localized in Uruk.

4. Linguistically indistinguishable from "Group 3". Its "provenience may likewise be Uruk".

5. Considered unspecifically northern by Goetze, and consisting of only three texts, one of which is a fragment and one heavily damaged. For terminological reasons, Eleanor Robson [2001: 183] proposes at least the third, YBC 6967, to belong to "Group 4"; but it shares as many terminological features with Haddad 104 (from Ešnunna, "7B"),[20] for which reasons the matter is best left pending.

6. Considered by Goetze to combine "northern and southern characteristics" and to be "slightly younger in date than the other groups". A footnote intimates a connection to Sippar, which has since then been corroborated and may now be considered fairly well-established.

7. Regularly excavated texts from Ešnunna. A subgroup "7A" consists of terminologically very similar texts found within neighbouring rooms; the remainder "7B" has no inner coherence and is only considered a "Group" for convenience. Most texts are found in dated contexts (1790 to 1775).

8. Regularly (but rather badly) excavated texts from Susa, probably of late Old Babylonian date.

Ur. Regularly excavated texts (but many found as fill) from 19th or early 18th-century Ur.

S. "Series texts", which Goetze did not consider because they contain almost no syllabic Akkadian. Neugebauer, who was the first to discuss the group [MKT I, 383*f*], proposed it to be from Kiš, but gave up the idea (as well as the term) in [MCT, 37]. They carry the name because the single tablets indicate in a colophon to be number so-and-so of a series.[21]

[20] In particular the results of calculations "coming up", cf. below, which they never do in "Group 4".

[21] Friberg [2000: 264] suggests to move the texts VAT 7528, YBC 4669, YBC 4698 and YBC 4673 ("Gruppe C" according to the division of the series texts in [MKT I, 506]) to a subgroup "2B" belonging together with "2A", the expurgated "group 2". Apart from the absence of serial numbering from the "2A" catalogues there are indeed outspoken similarities. These four texts also do not exhibit the complex organization of the other series texts described below. Incipient serialization was a general phenomenon in late Old Babylonian scribal culture; it is therefore quite possible that serialization of mathematics began independently in different places.

Problem formats and history

Taking into account a combination of external and internal criteria, we may construct a plausible scenario for the development of the Old Babylonian culture of mathematical problems.

The "Ur group" contains a few genuine problems only. Moreover, these exhibit no thematic intersection with what we find in the later Old Babylonian groups, and the problem format is rudimentary – a question en.nam (a.na.àm if an accusative is required) and an occasional ì.pàd.dè, "you will see" or a suffix .àm, "it is" indicating a result [Friberg 2000: 139–144, *passim*]. We seem to be at the watershed where a culture of problems is emerging, but still on the sole basis of the Ur III tradition.[22]

The earliest member of "Group 7", IM 55357 from c. 1790, already has a more developed structure.[23] After presenting the data it asks an explicit question; the prescription is introduced by the phrase za.e ak.ta.zu.un.dè, "You, to know the proceeding". Questions are asked by a syllabic *mīnum*, "what", or (in one place where an accusative is needed) a.na.àm.[24] Results are "seen", but the phrase is igi.dù (unorthographic for "open the eye", that is, "see"). The semantics is the same as in the "Ur group" and the Sargonic problems, but there is obviously no direct continuity at the terminological level. We must presume, either that already the Sargonic texts translate an Akkadian term (*tammar*, "you see"), or that Sumerian pàd has first been translated into and transmitted in Akkadian and then retranslated into Sumerian, the retranslator accidentally choosing a near-synonym.

The writing makes heavy use of logograms, for which reason it is impossible to ascertain whether the later systematic change of grammatical person (see imminently) was intended.

[22] The absence of a culture of mathematical problems in Ur III is dealt with in [Høyrup 2002c].

[23] Recently, Jöran Friberg and Farouk al-Rawi have published a volume [Friberg & al-Rawi 2016] containing a number of new texts from various localities from the Eshnunna Kingdom. If investigated from the present perspective, they would almost certainly add shades to the picture as presented here; circumstances have not allowed me to take up this task.

[24] This term, we remember, was also used in a single text from Ur. It seems never to turn up elsewhere. The outspoken differences in other respects seem to exclude that the Ešnunna text was inspired directly by what went on in Ur.

The texts from subgroup 7A, published in [Baqir 1951], share a new feature: an opening phrase *šumma kīam išal(-ka) umma šūma*, "If [somebody] asks (you) thus:" This refers to the typical opening of a riddle, and reveals an important source for the problem culture of the Old Babylonian scribe school – namely the professional riddles of mathematical practitioners (mostly but not exclusively surveyors). The statement itself is then mostly formulated in the first person singular ("I have [done so and so]").

The prescription opens with the formula *atta ina epēšika*, "you, by your proceeding" – close to that of the early text IM 55357, but now in syllabic Akkadian. Its "you" is followed up by use of the present tense, second person singular.

Often, the transition to a new section of the prescription is marked by the phrase *nasḫir*, "turn yourself around".

Results of calculations are marked by one of the phrases *tammar*, "you see", or *illiakkum*, "comes up for you" – in both cases often combined with an enclitic *-ma* on the verb for the operation.

A strange feature, with no analogue elsewhere in the corpus, is a coupling between interrogation and the announcement of results: when results "come up", the interrogative phrase of the question is *mīnum*, "what"; when they are "seen", we find *kī maṣi*, "corresponding to what". Possibly, two scribes with different habits were at work.

As stated, "7B" is no Group proper. Its eight members come from various locations – Tell Harmal, Tell Dhiba'i, and Tell Haddad. However, most of them open prescriptions by some variant of the phrase "You, by your proceeding" – one has a simple "You". Prescriptions carry the closing formula *kīam nēpešum*, "thus the procedure" (in contrast to Group 7A).

Two texts open as riddles, "if somebody ...". Haddad 104, containing 10 problems about topics rooted in Ur III practice, opens the statement *nēpeš*, "procedure of", or (if a variant is announced) *šumma*, "if (instead)". IM 52301 opens the statement *šumma*, the early IM 55357 by stating the object (saĝ.dù, "a triangle"), and IM 121613 by describing the situation.

Transitions to new sections may be marked by *tasaḫḫar*, "you turn around"; *tūr*, "turn back"; or as in "7A" *nasḫir*, "turn yourself around".

Results may be "seen", or they may "come up for you".

All in all, the Ešnunna texts reveal conscious attempts to create a problem format, but obviously no agreement about how this format should look; only "7A", presumably reflecting the ways of a single teacher or team of two teachers, has achieved something systematic. This, as well as the frequent riddle format, shows that we are confronted with the early phase of the development of a tradition[25] – which was then interrupted when Hammurabi conquered and destroyed the Ešnunna state in 1761. In spite of this, it is striking that most of the favourite themes of Old Babylonian mathematics are already dealt with.

Hammurabi *may* have brought Ešnunna scholars back to Babylon; the relation between the Ešnunna and the Hammurabi law codes indicate that at least he brought inspiration. Quite hypothetical is the possibility that he brought back teachers of mathematics. The Old Babylonian strata of Babylon are covered by later remains, which prevents us from finding any traces.

What we do know is that a mathematical problem culture turns up soon afterwards in the south. An important text belonging to "Group 1", the prism AO 8862, is doubtlessly related to a prism carrying tables in the Ur-III tradition (metrological tables and tables of squares, inverse squares and inverse cubes) that was written in Larsa in 1749 [Proust 2005]. Vacillating conventions (but mostly concerning the terminology for operations) both within this text (and within other texts from the same group) and between texts belonging to the group suggest that this group also reflects an incipient, not a mature tradition.[26]

[25] In a similar vein, Jean-Jacques Glassner [2005] uses the inhomogeneity of the technical terminology of haruspicy as evidence of a still immature discipline.

[26] A beginning around 1749 is contradicted by Eleanor Robson's dating [2001: 172] of the tablet Plimpton 322 (which she supposes to be from Larsa) to "the 60 years or so before the siege and capture of Larsa by Hammurabi of Babylon in 1762 BCE", Her argument, however, is far from coercive. She observes that the tablet is in landscape format, and that this format was used in the Larsa *bureaucracy* from 1822 onward. However, the contents of the text – a table with many columns – asks for this format. Even if it had gone out of administrative fashion after the conquest, it would be an obvious choice to use it when it was adequate (also outside Larsa, for that matter). The particular southern spelling of the texts of the group show that they were written by scribes who had received their education locally. In Goetze's words, the "Plimpton tablet may or may not belong" together with the indubitable Larsa texts (it contains very few words).

As a rule, the texts belonging to the group open by stating either the object or the situation. The prescription normally opens with an Akkadian syllabic "you, by your procedure" or "by your procedure"; there is no closing formula. Results are mostly marked by nothing but an enclitic *–ma* on the preceding verb (Akkadian as well as Sumerian being verb-final, the verb always closes the operation); occasionally, however, they "come up".

The riddle introduction has disappeared. In Groups 2–6 and 8, where it is also absent, the system of two voices is reinterpreted, and the statement stands out as if is was formulated by the master telling the situation "I" have produced, while the prescription is formulated by the instructor or "elder brother"[27] in the second person singular or the imperative, at times arguing for a particular step with an exact quotation of what "he" (the master) has said. This system is still only imperfectly present in Group 1, where the prescription may shift between what "you" shall do and what "I" do, and where results sometimes come up "for me" and sometimes "for you" (regularly within the same text). This fits an incipient, still not firmly established tradition.

Striking is the absence of *tammar*, "you see", not only from this group but also from the other southern Groups (2–4). Since this term was characteristic of the Ešnunna texts and presumably of the Akkadian lay (non-scribal) tradition, avoiding it[28] may have been a way to demarcate oneself from the conqueror.[29]

The core of "Group 2A" (the expurgated "group 2") is constituted by two theme texts about "excavations", to which come a number of statement catalogues

[27] This šeš.gal is a familiar figure from the *edubba* literature. Cf., e.g., [Kramer 1949, *passim*].

[28] An oblique reference in the "group 3" text YBC 4608 (a question what to do *aš-šu x a-ma-ri-i-ka*, "in order that you see *x*", shows that the idiom was known. YBC 4662, belonging to "Group 2A", also has a single isolated *tammar*. The almost complete but not total absence of *tammar* must thus reflect a conscious effort to avoid it.

[29] This argument does not presuppose any kind of patriotic feelings, which may or may not have existed. A local elite will automatically resent coming under control of foreigners and thus to descend the hierarchical ladder – as pointed out sharply by Samsî-Addu to his son Yasmah-Addu deputy king of Mari when the latter had expressed the intention to give official functions to captive nobles from Ešnunna [Durand 1997: I, 182*f*].This was probably more than the mere suspicion of a cautious and shrewd ruler. Michel Tanret [2010: 247] points to a symbolic act of resistance on the part of a temple manager in Sippar against the Babylonian conqueror.

without prescription – in part containing the statements of the theme texts, and thus certainly coming from the same locality and school. The statements (of theme texts as well as catalogues) are heavily logographic, the prescriptions of the theme texts predominantly syllabic.

The statements start by announcing the situation, and then ask a question marked en.nam, "what" (in a single case *kī maṣi*, "corresponding to what"). The logographic phrase za.e kid$_9$/kid.da.zu.de,[30] "you, by your making", serves to open the prescription. In one of the theme texts it closes *kīam nēpešum*. As a rule, results "come up for you" (but as mentioned, the theme text YBC 4662 contains a single *tammar*).

Many problems in the two theme texts combine the determination of the geometric object (which may constitute a directly geometric or an "algebraic" problem[31]) with a calculation of the wages to be paid, thus with a normal scribal concern. This, as well as the format, suggests that the texts of this group constitute a direct continuation of the normal mathematical curriculum of the scribe school.

The linguistically indistinguishable "Group 3" and "Group 4" are probably both from Uruk. None the less, they are different in their choices of format and even more as terminology is concerned – so different that one may suspect deliberate demarcation. Internally, each group is rather coherent.

In "Group 3", the statement is an unadorned presentation of the situation, ending with a question (mostly marked en.nam, more rarely *kī maṣi*, once in a problem about the distribution between brothers *kiyā*[32]). If a prescription is present, it opens with the phrase za.e kìd.da.zu.de. There is no closing formula.

[30] Both unorthographic, which (like the isolated occurrence of *tammar* in YBC 4662) is perhaps evidence that these texts contain rewritten northern material – orthographic writing would have employed kìd.

[31] I shall abstain from taking up the question whether Old Babylonian "algebra" is justly characterized as an algebra or not, which others find much more interesting than I do, and the answer to which depends on definitions and taste. Examination of the discipline in question (which I shall go on referring to in quotes) is the main topic of [Høyrup 2002a] as well as [Høyrup 2017].

[32] In the Old Babylonian corpus, this is the normal way in all groups to ask for several values, and it may thus adequately be translated "how much each". Non-mathematical contexts appear not to require this plurality [CAD 8, 329a].

Results of any kind are followed by a logographic sum, "it gives" – except in four passages, where it is syllabic. Three are instances of the "division question", "what shall I posit to *P* which gives me *Q*?"; the syllabic writing thus serves to make clear that a subjunctive is meant.

The only "logical operator" appearing in the group is *aššum*, "since"; it introduces an argument by "single false position" in VAT 7532 and VAT 7535, "since $^1/_6$ of the original reed was broken off, inscribe 6, let 1 go away, ...".

"Group 4" also opens statements by describing the situation – occasionally defining the object first; the question is made explicit, mostly by en.nam (in a peripheral subgroup by a syllabic *mīnum*), more rarely by *kī maṣi*, and in one "brother problem" by *kiyā*. In a few cases the prescription starts by *atta*, "you", but mostly there is no opening phrase, as there is no closing formula.

Results are mostly marked by a preceding enclitic *-ma*. Syllabic writings of *nadānum*, "to give", are mostly used in connection with the "division question", but on a few occasions for the outcome of "raising" multiplications.

The logical operator *šumma*, "if", is used regularly, sometimes in the beginning of statements regarded as variants (which excludes its being a remnant of the "riddle opening"), more often in the beginning of final verifications, which are frequent in this group. In three texts it is used within the prescription to open a new line of reasoning after a preliminary result has been established. *aššum*, "since", is used to introduce quotations from the statement, and furthermore once in a broken, incomprehensible passage (VAT 8523, rev. 8).

In contrast to "Group 1", the two Uruk groups look as if they represent already settled local traditions. Uruk and Larsa being separated by less than 25 km, it is rather unlikely that they can have been produced before a "Group 1" still groping for a canonical style.[33] A date after c. 1745 seems inherently more plausible.

At the same time, it is virtually certain that all southern texts (groups 1–4) were produced before 1720 – after the successful secession of the Sealand there

[33] A referee directs attention to CBM 12648 (in recent years CBS 12648), a mutilated tablet from Nippur, written in hypercorrect Sumerian, probably before 1900. What survives is a volume calculation on the obverse [Friberg 2001:149*f*] and, on the even more damaged reverse [Robson 2000: 32], remains of a problem format similar to what we know from Ur from the same period (a question en.nam and an enclitic .bi, "its"). There is no continuity with what we see in the later southern text groups; if anything, this text shows that the Larsa and Uruk groups represent a fresh beginning.

seems to have been a violent decline in literate culture in the area. That would leave at most 30 years for the creation of the southern text Groups – a single or at most two generations of workers. That may seem problematic until we discover that these groups present us with only one substantial innovation as compared to what we know from Ešnunna: the idea of *representation*, such as use of the lengths and widths of the "algebraic" rectangles as representatives for a number of workers and the number of days they work. The other innovations consist in the creation of variants and, in particular, of *canonical styles*. That could easily be achieved within a single generation.

Whether the three texts counted as "Group 5" really form a group is uncertain, as is its localization in the North – cf. above, before note 20. In any case, the "Group" is too small to tell us very much. "Group 6" is much more interesting.

It is certainly northern, and in all probability to be located in Sippar. In all probability it is also later than the southern groups. To its core ("6A") belong a number of procedure texts containing many problems, one (BM 85200 + VAT 6599) strictly dealing with "excavations" (see above around note 15), others (BM 85194, BM 85196, BM 85210) either "theme texts" with a very liberal idea of how to delimit the theme ("geometrical calculation of anything"?) or outright mathematical anthologies.

As a rule, "you see" results in this Group, which shows it not to descend from the southern groups but to be a later member of the same extended family as the Ešnunna texts.

Very often, statements start by defining the object. Sometimes, however, this is omitted, and we get a description of the situation (often neutral, but at times told in the first person singular). In a few cases, mostly not concerning variants, the beginning is *šumma*, "if". This *might* be a remnant of the riddle opening, but nothing else in the texts supports such a connection. On a few occasions, the statement is supported by an explanatory diagram. The question is normally asked with en.nam, very rarely with *kī maṣi*. Prescriptions open za.e, "you", and close *nēpešum*, "the procedure", occasionally *kīam nēpešum*, "thus the procedure".

Beyond *šumma*, the logical operators *inūma*, "as", and *aššum*, "since", both turn up a few times, the former to introduce a small piece of embedded reasoning, the second probably with the same function (but all relevant passages are strongly damaged).

kīam nēpešum, "thus the procedure", was also used in "7B" as the closing formula. It is totally absent from the southern texts, also in the abbreviated form *nēpešum*. This corroborates the conclusion derived from the use of *tammar*, namely that "Group 6" belongs to the same family as the Ešnunna texts. There is no reason to believe that its style was borrowed from scholars who had gone north, emigrating from the Sealand (as Goetze had believed).

The "series texts", on the other hand, or at least some of them, may be in debt to southern scholars, even though their almost certainly late date[34] tells us that they must have been produced in the North.

As stated above, the single tablets indicate their number within a series; partial overlaps etc. shows that several such series existed, and that there is no trace of "canonization".

The texts contain only problem statements (and sometimes a numerical answer). They are written in a very compact and highly stylized logographic notation – even prepositions are replaced by Sumerian case endings, but nonetheless the language is even farther from being Sumerian than Akkadian.[35]

Within the single statements, there is no problem format apart from a facultative en.nam specifying the question. Globally, however, only the existence of a strict format allowed the users of the text (and allows us, when we are lucky!) to understand the situation that is delineated. As an example, we may look at the translation of a sequence from YBC 4668 (following [Høyrup 2002a: 201*f*]). Round brackets (...) are explanations that are needed for minimal comprehensibility, pointed brackets <...> indicate words that according to the general style of the text should have been there but are none the less omitted.

[34] Firstly, the utterly compact formulation of these texts must be the outcome of a long development; secondly, serialization seems in general to have taken its beginning in the final Old Babylonian century. Thirdly, Christine Proust [2015: 284, cf. 2009: 195] argues from the structure of the colophons for links to "a tradition which developed in Sippar at the end of the dynasty of Hammurabi".

[35] Neugebauer [1934: 70–72] compares this compact logographic writing to an algebraic symbolism, though explaining how this has to be understood in order to be adequate – actually, his interpretation looks more like the description of an algorism than as a manipulation of symbols. Later general histories of mathematics have sometimes been too eager to claim the algebraic symbolism without caring for Neugebauer's restrictive use of the term.

Rev. III

#34	4.	The surface, 1 eše
	5.	The fraction, of the width, concerning the length
	6.	to the length raised, 45.
	7.	The fraction, of the length, concerning the width,
	8.	<to> the width raised: 13° 20′
	9.	its length, width what?
...		
#38	19.	The 19th part (the excess) of (that) which to the length (is) raised
	20.	over (that which to) the width (is) raised, goes beyond
	21.	(to that) which to the length (is) raised, appended, 46° 40′.
#39	22.	(In) steps 2 repeated, appended, 48° 20.
#40	23.	Torn out: 43° 20′.
#41	24.	(In) steps 2 repeated, torn out, 41° 40′.
#42	25.	(To that) which to the width (is) raised, appended: 15.
#43	26.	(In) steps 2 repeated, appended: 16° 40′.
#44	27.	Torn out: 11° 40′.
#45	28.	(In) steps 2 repeated, torn out, 10.
#46	29.	The surface, 1 eše
	30.	The 7th part of (that) which (to) the length, (of the) width, (is) raised,
	31.	(and that) which (to) the width, (of the) length, appended, 53° 20′.
#47	32.	(In) steps 2 repeated: 1` 1° 40′.
#48	33.	Torn out: 36° 40′.
#49	34.	(In) steps 2, torn out, 28° 20′.
#50	35.	(To that) which to the width (is) raised,
	36.	appended: 21` 40′.
#51	37.	(In) steps 2 repeated, appended: 30.
#52	38.	Torn out: 5.
#53	39.	(In) steps 2 repeated:
	40.	3° 20′ it went beyond.

All problems deal with a rectangle $\sqsubset\!\sqsupset(l,w)$. In #34, we are told that the area $\sqsubset\!\sqsupset(l,w)$ is 1 eše, that is, 600 times the square on the basic length unit. Further, $L = {}^{l}/_{w}\cdot l = 45$, $W = {}^{w}/_{l}\cdot w = 13°20′$. Since $\sqsubset\!\sqsupset(L,W) = (l,w)$, this is a standard problem, just embedded in a rather trivial complication.[36] In #38, the area is presupposed to be unchanged (whence not mentioned); the other condition is ${}^{1}/_{19}\cdot(L-W)+L = 46°40′$. The single line in #39 means that the second condition is now ${}^{2}/_{19}\cdot(L-W)+L = 48°20′$, while that of #40 is $L-{}^{1}/_{19}\cdot(L-W) = 43°20′$. In #42, it becomes ${}^{1}/_{19}\cdot(L-W)+W = 15$, while #46 changes the denominator from 19 into 7. We thus have exploration of all the possibilities obtained by changing the second condition along 4 dimensions.

Grammatical, medical and extispicy lists also attempt to be systematic, and sometimes we find sequences which vary along two dimensions – but not more, and never as perfectly as here, for the simple reason that the subject-matter does not allow it.[37] Within mathematics, we also have nothing coming close, neither earlier nor later. Even more obviously than the Old Babylonian problem culture in general, the series text represent a species that was too highly specialized to survive the particular environment where it had emerged.

We know about selective adoption/adaption of Mesopotamian metrology and tables outside the Babylonian area in the second and first millennium, but the only place where we have evidence of a broad adoption of the *problem culture* is in (probably late Old Babylonian) Susa.

The texts contained in the volume [TMS] are evidence of that – more precisely "group 8A" consisting of the procedure texts TMS VII–XXV.[38] For the present purpose the most important observation to make is that results are marked *tammar*, "you see". Statements open by describing the situation which "I" have created. The prescription opens with a simple *atta*, "you", and ends (except in the two texts that explain that this was the "bundling" or "Akkadian" method, cf. above) just by pointing out that the number resulting from the last calculation is the quantity asked for.

[36] Once L and W are found, we have to use that ${}^{L}/_{W} = ({}^{l}/_{w})^3$. Knowing ${}^{l}/_{w}$ and $\sqsubset\!\sqsupset(l,w)$ we find $\square(l)$ as their product, whence also l – etc.

[37] Nor are all series texts as systematic as this passage. The variations in for instance YBC 4714 [Høyrup 2002a: 112–132] are no more orderly than those of well-structured medical texts.

[38] TMS V–VI, "group 8B", are two statement catalogues, "8C" a single atypical procedure text. TMS I–IV are tables and drawings of polygons with numbers written into them.

The appearance of *tammar* shows that the Susa texts belong to the same broad area as the Ešnunna and Sippar-texts. Other characteristics make it clear that they do not descend directly from any of these particular groups. If it is true, as argued above, that the southern traditions were only established after c. 1750 and vanished before 1720, we should perhaps not wonder that they did not leave a strong impact in the following century. The good luck of excavators – that cities burn or are left in haste – is not necessarily the best for the influence of scholarly traditions.

Names for objects

There is no reason to discuss the names for objects with respect to the single text groups – to a large extent they are used transversally. I shall restrict myself to two observations.

One of these has to do with a tendency to apply "default understanding". If a problem statement presents its object as uš saĝ, "length width", it does not deal with a length and a width but with a figure characterized by possessing a length and a width – and moreover, by the simplest figure (as seen by the Babylonians) which is characterized by possessing them, that is, a rectangle. And when Db_2–146 starts *šum ma-ṣi-li-ip-ta-a-am-i-ša-lu-ka*, "if about a cross-over (somebody) asks you", the meaning is that he asks about the simplest possible configuration possessing a cross-over (i.e., diagonal).

Both expressions reflect the fundamental way of the Babylonians to think of the objects of their mathematics – namely "by default".

This does not correspond to what we believe about our own thinking – but perhaps we are wrong, and perhaps we are more Babylonian than we recognize. In languages where the counterparts of "quadrilateral" (German *Viereck*, Danish *firkant*, and even French *quadrilatère*, Arabic *murabbaᶜ*) belong to current non-technical speech, they are often used in the more specific sense of *square* (in Arabic even primarily). Before the monster-hunt of nineteenth-century mathematicians, a *function* was also presupposed to be not only continuous but also smooth. If it was not, that had to be made explicit. And much of the argument in Lakatos's *Proofs and Refutations* [1976] is indeed built up around objects that are gradually discovered not to possess necessarily the properties (convexity etc.) that were presupposed by default.

The other observation to make has to do with syllabic versus orthographic writings of the terms for "length" and "width". As we have already seen, the

"you" introducing a prescription is written za.e in some text groups and *atta* in others. Many terms for operations behave similarly, or they are written logographically in straight and syllabically in oblique forms (for instance, the subjunctive and the precative); a logographic term in a statement may even be quoted syllabically as what "he" has said.

However, the case of "length" and "width" is different. We may start by observing that they occur in two different roles. They may be the extensions of *real* geometric objects – a carrying distance, the length of a wall, or the dimensions of a real field. In that case they may be written either way, as *šiddum*/uš ("length") respectively *pūtum*/saĝ ("width"). But they may also be the dimensions of the abstract rectangles used as a basic representation in the "algebra", and then they are invariably written logographically, and without any grammatical complement that might indicate an Akkadian pronunciation – *except* in a few texts from Ešnunna and two texts plausibly from early Sippar.[39] It thus seems that a firm conceptual distinction between real distances and the "abstract" extensions used in "algebraic" representation was only establishing itself around 1775. In all later text groups its presence is subject to no doubt.

Operations

Even the terminology for operations need not be systematically discussed with respect to the single groups – this would not yield much further information, nor contradict the results already obtained. Instead, a list of operations and corresponding terms will do, with observations about their occurrence when such are called for.

Additive operations

One addition consists in joining one magnitude d to another one A. In this process, A conserves its identity but increases in magnitude; the sum thus has no name of its own. The operation is concrete, and d and A must by necessity be of the same kind. The Old Babylonian term for this operation is *waṣābum* (I use the standard translation "to append"). In two texts from the disparate "group 1" (YBC 6504 and AO 6770), in "Group 3", "Group 6", and in the series and Susa texts it may be replaced by

[39] The "Tell Harmal Compendium" (IM 52916+52685+52304), Db_2-146 and IM 43993; and CBS 43 and CBS 154+921, which indicate syllabic possessive suffices, cf. below, note 53.

daḫ (sometimes with grammatical complements). The word is regular Sumerian, meaning "to add, to say further, to help" [Thomsen 1984: 298]; the idea to use it as a logogram for *waṣābum* seems to be secondary.[40] The inverse operation of this addition is subtraction by removal, see presently.

Another addition is *kamārum*, "to accumulate" (or "to heap"). It is symmetric, and dissolves the two addends into a common sum (*nakmartum* or, if still understood as the plurality of constituent parts, the plural *kimrātum* of *kimirtum*).[41] It may be used for the formal addition of quantities of different kinds, in which case the addition concerns the measuring numbers of the quantities involved. The logogram ĝár.ĝár appears to be of genuine Sumerian origin, cf. above, after note 11. "Group 6" and the Susa texts instead use UL.GAR, which is unexplained. The rarely used inverse of this addition is separation into constituent components (*bêrum*).

In some early "algebraic" texts (from Ešnunna and "Group 1"), sides of rectangles and squares are "appended" to areas, which implies that they are regarded as "broad lines", provided with a standard width equal to one linear unit.[42] These appear to have been eliminated in the same process as established mature problem formats. Afterwards these additions were always formulated as "accumulations".

Subtractive operations

There are two subtractive operations, but a whole gamut of terms for them. One is *removal*, the inverse of "appending" and equally identity-conserving; the entity that is removed has to be a part of the one from which it is removed. The main terms for this are *nasāḫum*, "to tear out", and *ḫarāṣum*, "to cut off". The latter is mostly used in the Ešnunna texts and in "group 1". It may perhaps have been the preferred term of the lay surveyors, who provided the basis on which the "algebraic"

[40] A lexical list states daḫ to be the equivalent of *ruddûm* – whose meaning is "to add (numbers, silver, commodities, goods, immovable property), to add words, entries in a tablet, to add a statement" (<*redûm*) [CAD 14, 226*f*]. This word seems never to appear as a mathematical term.

[41] Readers who are familiar with the structure of classical Semitic languages will notice that all are derived from the root KMR.

[42] The frequent appearance of this conceptualization of lines (which allows lengths and areas to be measured in the same units) in pre-Modern practical geometries is discussed in [Høyrup 1995].

discipline was developed; this assumption would agree with the absence of a corresponding logogram. The former, which replaced it as the normal term for the operation (but is already found in three texts from Ešnunna), was provided with the semantically improper logogram zi, cf. above, after note 11.

In situations where connotations suggest a different metaphor, other terms for removal turn up occasionally. One example was quoted above for a different purpose, namely "since $^1/_6$ of the original reed was broken off, inscribe 6, let 1 go away, ..." (VAT 7532, VAT 7535). "Let go away" translates *šutbûm* (<*tebûm*). In non-mathematical contexts, this verb is regularly used for removing something that *should* go away, which is exactly the case in these false-position arguments.

Another rare term for removal is *tabālum*. It occurs in the Susa problem catalogue TMS V, section 12, in which a part of an area is "withdrawn"; since this "area" might also be a real field, the problems could very well deal with the real-life situation where this part has been *withdrawn by legal action*, as is the normal use of the term.[43] A second occurence is in the text YBC 4608 (obv. 24), where a line a is withdrawn from what is already known to represent the sum $a+b$ of two opposite sides of a quadrangle. Probably, the term is chosen here because of the connotation of something which is due to be done.

That connotations played a role is confirmed by those texts which employ *ḫarāṣum* and *nasāḫum* together (in particular AO 8862): they tend to "cut off" from lines and "tear out" from surfaces.

The other subtractive operation is *comparison* of different entities. Most often, it is made by the phrase *A eli B d itter/īter*, "*A* over *B*, *d* it goes/went beyond" (from *eli ... watārum*, "go beyond", "be(come)/make greater than"),[44] with the Sumerographic equivalent *A* ugu *B d* dirig. In the Susa texts dirig also serves as a logogram for the excess, that is, for that amount *d* by which *A* "goes beyond" *B*.

[43] This interpretation fits the fact that these statements are in the third, not the first person singular. It is not the teacher who is supposed to have performed the action, as in the other statements.

[44] If we give up the ambition to render the grammatical structure of the Akkadian phrase, we may also translate "*A* exceeds *B* by *d*".

However, various reasons may determine that the comparison is made the other way around.[45] Then the text does not say by how much A exceeds B but instead by how much B falls short of A, using the verb *maṭûm*, "to be(come) small(er)" (Sumerogram lal).

"Multiplications"

Three genuine multiplicative operations can be found in the Old Babylonian texts. One is a.rá, "steps of", the multiplication of number by number. It is the term of the multiplication tables (including the table of squares); in problem texts it is very rare.[46]

The second multiplicative operation is *našûm*/íl (occasionally nim), "to raise". Its origin is in volume calculation, and it refers to the "raising" of the base from the default height of 1 cubit to the real height; from there it was transferred to other multiplicative determinations of concrete magnitudes.[47] In particular, it is always used in multiplications by technical (igi.gub) constants and by reciprocals. We have no evidence that it was already used during Ur III, but on the other hand we have no texts where we would expect it to turn up. Since the result of a "raising" is often stated to be "given", also in groups that do not use this term for resulting in general, it is at least likely to belong together with the complex of place-value computation (cf. above, after note 12).

[45] The systematic structure of series texts may be one such reason; another is the aspiration that relative differences should be one of the "favourite fractions" ($^1/_4$, $^1/_7$, $^1/_{14}$, etc.) and not for instance $^1/_6$ or $^1/_8$ [Høyrup 1993]; finally, the stylistic habit to take the outcome of one calculation as the subject of the next sentence may require that this outcome be said to fall short of another quantity.

[46] Two of the problem texts from Ur, UET 5,864 and UET 5,858, have the phrase *a* a.rá *b* ù.ub.rá, "*a* steps of *b*, when you go". AO 8862 ("group 1", cf. above) uses *a* a.rá *b* repeatedly where we would expect a "holding" or (once) an ordinary halving (for both operations, see below). The two theme texts from "Group 2A" (YBC 4662, 4663) apply it a few times, in the phrases *a* a.rá *b iši*, "*a* steps of *b* raise", or *a* a.rá *b* UR.UR.A, "*a* steps of *b* make hold". The atypical Susa text TMS XXVI [Muroi 2001: 229*f*] has the purely numerical sequences 26,40 a.rá 2 53,20 – 35 a.rá 35 20,25 – 1,20 a.rá 6 8 – and the sequential 1,20 a.rá 20 26,40 a.rá 2,53,20. Finally, some series texts (e.g., YBC 4668) couple it to "repeating" (see below), with phrases like a.rá 2 e.tab, "(in) two steps repeated".

[47] That volume determination is the origin can be seen by the order of factors. When volumes are concerned, it is invariably the base that is "raised" to the height. In all other situations, the order is determined by the textual structure, the number which has just been found being "raised" to the other factor.

The third multiplication is "repeating". We have encountered it above, as one of the possible ways to express the circumference of the circle in terms of the diameter (above, after note 2). The main term is *eṣēpum*/tab,[48] with nim as an occasional alternative. When occurring without a specification "to n", its meaning is doubling. Except in a few instances in the series texts, n is always smaller than 10, and the term always refers to a concrete n-doubling of the tangible entity concerned, not to a mere numerical multiplication.

In the Susa corpus (TMS VII, VIII), syllabic forms of *alākum*, "to go" (until n), occur both as equivalents of *eṣēpum* and when an "appending" is to be repeated. This reveals an underlying conceptual connection between the operations of "steps" and "repetition", as also confirmed by the Ur occurrences of the phrase *a* a.rá *b* ù.ub.rá, "*a* steps of *b*, when you go" and of the use of certain series texts of the phrase a.rá *n* e.tab, "(in) *n* steps repeated" – cf. note 46.

Rectangularization and squaring

A term which is traditionally also translated as "multiplication" is *šutakūlum*, with a number of logographic equivalents. Actually, it stands for the construction of a rectangle with sides *a* and *b*. As a rule, the calculation of the area is understood to be implied in the process, but if the rectangle is already there, its area is found by "raising", showing that *šutakūlum* cannot be a mere area determination.

The verb is the causative-reciprocative form ("make ... each other" or "make ... together"), either of *akālum*, "to eat" (the guess of Neugebauer), or of *kullum*, "to hold" (that of François Thureau-Dangin). Since that which has been caused to "eat"/"hold" can either be referred to by the relative phrase *ša tuštakil* or by the noun *takīltum*, which can only be derived from *kullum*, there is now no doubt that Thureau-Dangin was right;[49] moreover, since the double object (the two segments

[48] The basic meaning of tab being "to be/make double, to clutch, to clasp to" [Thomsen 1984: 318], the logogram is obviously not very adequate but a secondary choice, derived from one of the meanings of the Akkadian term.

[49] An apparent counter-argument is the use of the logogram $gu_7.gu_7$, "eat-eat". However, Sumerian reduplication did not correspond to Akkadian causative-reciprocative, and the logogram is thus clearly a secondary construction, formed from the Akkadian (as are the other reduplicated logograms, cf. below), and such a secondary construction could easily be inspired by the quasi-coincidence of the corresponding forms of *kullum* (*šutakūlum* or possibly *šutakūllum*) respectively *akālum* (*šutākulum*) – such puns or rebus-writings had been the fundament for the whole development of cuneiform writing from the purely logographic-pictographic script of the fourth millennium.

a and *b* that are "caused to hold") are sometimes connected by the preposition *itti*, "together with", the meaning must be "make *a* and *b* hold together". Even though there is no reason to assume semantic continuity (nor to exclude it), the idea is thus the same as in Greek geometry: even here, a rectangle is "contained" or "held" (περιέχω) by two sides (*Elements* II, def. 1 [ed. Heiberg 1883: I, 118]).

The construction of a square with side *a* may be described by the same term (either "making *a* and *a* hold" or just "making *a* hold"); but it may also be spoken of with the equally causative-reciprocative *šutamḫurum*, "to make (*a*) confront itself", derived from *maḫārum*, "to confront (on a footing of equality)". To this corresponds the term *mitḫartum* for the square configuration (literally something like "a situation characterized by the confrontation of equals"). Unexpectedly for us but in good agreement with the meaning of the word (which refers to the square frame, not to the area it contains), the numerical value of the *mitḫartum* is the length of the side – a *mitḫartum is* its side and *has* an area, while our square *has* a side and *is* an area.[50] If one side of a square has been found, the other side meeting it in a corner is referred to as its *meḫrum*, "counterpart".

Both *šutakūlum* and *šutamḫurum* have logographic equivalents, but most of these can stand for either of the Akkadian terms. gu$_7$.gu$_7$ was mentioned in note 49. Beyond that, there is UL.UL, almost certainly to be read du$_7$.du$_7$, properly "to butt each other" but according to backward syllabic references in relative phrases actually to be read *šutakūlum*; UR.UR – no certain explanation seems to be at hand, but cf. note 46; LAGAB, whose sign is a square frame, and which may be iconic, and LAGAB.LAGAB = NIGIN, which *may* combine the iconic aspect of LAGAB with the causative-reciprocative aspect of the reduplication. Because of the imperfect correspondence with the two Akkadian words, it may be better to see all these terms as *ideographic* (in the sense our mathematical symbols like "+" are ideographic) and not as genuine logograms.

The side of a square area (corresponding in modern but inadequate terms to the square root) is mostly expressed by the terms íb. si$_8$, ba.si$_8$ or (in Ešnunna) some unorthographic variant. In Ešnunna, the ba-variants are sometimes preferred, elsewhere (as a rule) these are reserved for cubic and quasi-cubic sides. This *may* but need not have to do with the different ways in which Ešnunna and the South inherited the Ur III tradition.

[50] This is the real background to the nonsensical claim sometimes advanced, that the Babylonians did not distinguish a square from a square root.

The full phrase of the inverse square tables is *A*.e *s* íb.si$_8$ (cf. note 12), si$_8$ meaning "to be equal". The final position of íb. si$_8$ shows this to be meant as a verb; the grammatical case of interrogatives shows that the interpretation in the Ur group was "close by *A*, *s* is equal", while all later groups that conserve the verb interpretation appear to have changed the reading into "*A* causes *s* to be equal".[51] A translation that renders both is "by *A*, *s* is equal".[52]

However, not all text groups understand íb. si$_8$ as a verb. "Group 7A" does, but most texts from "Group 7B" understand it as a noun; this "equalside" was probably thought of as the kind of "thing" listed in the tables. "Group 1" is also uneven in its usage, while groups "2A" and "ii" opt for the verb. So does "Group 3", while "Group 4", the other Uruk group, and the single pertinent text from "group 5" opt for the noun. "Group 6" mostly asks and answers with the verb, but sometimes falls entirely outside the pattern, and states in syllabic Akkadian that *s imtaḫḫar*, "*s* confronts itself", *s* ta.àm *imtaḫḫar*, "*s*, each, confronts itself", or *s* íb. si$_8$ *imtaḫḫar*, "*s*, as equalside, confronts itself".[53]

Division and parts

As is well known, division was no operation in sexagesimal place-value arithmetic. Division *problems* were of course well known (also in practical computation). If possible, the problem was solved via multiplication with the reciprocal; in practical computation this could always be done, since those technical constants which might

[51] The Sumerian suffix .e may be terminative-locative as well as agentive – cf. English "by".

[52] Quite unique in the corpus, YBC 6504 (an outlier in "group 1") uses íb.si$_8$ in two of four parallel passages for squaring, presumably for *šutamḫurum*, and du$_7$.du$_7$ in the others. The geometric text BM 15285 uses íb.si$_8$ logographically for *mitḫartum* meant as a geometric configuration.

[53] The phrase *a imtaḫḫar* is also found in BM 13901 #23, a problem that conspicuously leaves the canonical formulations of this long texts about squares and quotes a traditional riddle of the lay surveyors in their characteristic parlance – cf. [Høyrup 2002a: 222–226]. There is nothing jocular about the "Group 6" texts; their use of the same phrase thus points to genuine vicinity to the same environment. The question *kiyā imtaḫḫar*, "how much, each, stands against itself", making even more clear that several sides are asked for, is found in the related texts CBS 43 and CBS 154+921 [ed. Robson 2000: 39*f*]. These texts are unprovenanced (because of too swift reading of Eleanor Robson's publication I ascribed them to Nippur in [Høyrup 2002a: 354]). However, the writing of uš with a phonetic grammatical complement ^{ia}, "my", suggests them to be early, probably contemporary with the Ešnunna texts; Robson tells me (personal communication) that they *may* be from Sippar – but they obviously do not belong to "group 6".

turn up as divisors were always chosen so as to possess a simple sexagesimal reciprocal. In mathematical school texts, however, many division questions appear that cannot be solved in this way. Then the division question "what shall I posit to *P* which gives me *Q*?" is asked, and the answer stated immediately. Since the problems where it happens were invariably constructed backwards from known solutions, the answer would always exist and always be known to the author of the problem.

This is the case in almost all text groups – the exceptions being the Ur group, where the formulation in UET 5,859 is somewhat different, and the series texts, where no prescriptions are present and the questions therefore do not arise. There is no reason to elaborate.

It is also well known, but not much spoken about, that the expression igi *n* may as well refer to the reciprocal of *n* as to the *n*th part *of something*.

Originally, there was no difference. As shown by Piotr Steinkeller [1979: 187], some early tables of reciprocals (mostly of Ur III date) make clear that they list not reciprocals in our sense but *n*th parts of 60 – an example is published in [Oelsner 2001: 56]. Obviously, that makes no difference in the numbers when written in a floating-point system, and in Old Babylonian times the reciprocal and the *n*th part were clearly distinct concepts.

There was no standard way to keep the two ideas clear of each other; all the more interesting is it that different texts, though using different verbal means, distinguish them very clearly.

The basic term for the reciprocal of *n* is igi *n* ĝál.bi, "[of 1], its igi *n* ĝál", whose meaning is enigmatic. igi, originally the picture of an eye, is used as a logogram for *īnum*, "eye", for *amārum*, "to see", and for *pānum*, "face". The latter gave rise to an Old Babylonian folk etymology, the igi of a number being either replaced by or glossed as *pāni*,[54] "in front of", namely "is placed (ĝál) in front of *n* in the table of reciprocals". However, the use of igi for "part" goes back at least to the early 24th century, thus antedating the tables of reciprocals by 300 years or more. The only plausible explanation (whose central idea goes back to [Friberg 1978: 45]) is that the phrase means "*n* placed in eye", which would be a description of the proto-literate notation for fractions in the grain system [Damerow & Englund 1987: 136]. Since half a millennium without any fractions in the record separates the two

[54] Replaced in Haddad 104, glossed in the "Group 6" text BM 96957.

notations, this can be nothing but a hypothesis.

Many tables of reciprocals carry the full phrase igi *n* ĝál.bi, others abbreviate it into igi *n* ĝál. or igi *n*. Tables evidently do not speak about the *n*th part of something (except in the sense just mentioned); in order to see the distinction we must look at problem texts that refer both to reciprocals and to parts.

One possibility is ellipsis. The "Group 3" texts Str 367, VAT 7532, VAT 7535, etc. speak of the *n*th part of something (even of 1 if this number represents an unknown length in an argument by false position) by the phrase igi *n* ĝál; the number facing *n* in the table of reciprocals is simply igi *n*. In the text BM 85210 ("Group 6") the same distinction is made, but supplemented by the use of different verbs: the reciprocal is "detached" (du_8), as it always is; the *n*th part of *m*, however, is "torn out" (zi). BM 85194 (also "Group 6") uses the short form for both concepts, and distinguishes by the choice of verb alone.

Halves and halving

Old Babylonian mathematics distinguishes two "halves". One belongs to the same general class as $^1/_3$, $^1/_4$, etc. This half may be a number (30′) or the half of something. It can be written syllabically (*mišlum*); as 30′; with the sign BAR (+); or with the Sumerogram šu.ri.a.

The other is a "natural half", invariably *of something*. It is mostly spoken of as *bāmtum* (in general language "half-share", one of two opposite mountain-ridge slopes or body parts),[55] but in Db_2-146 ("Group 6B") it appears as *muttatum* (generally "half-pack" etc.). It is used in situations where no fraction but the half would do – the radius as half of the diameter, the half of the base of a triangle serving in area computation, etc. It has no proper logogram, but the strongly logographic text YBC 6504 (the "outlier text" from "group 1") use šu.ri.a, while groups "3" and "6" as well as the Susa texts sometimes or always use BAR.

The operation by which a natural half is produced is "to break" (*ḫepûm*/gaz).[56] *ḫepûm* as well as gaz have the general meaning "to smash", "to destroy", "to

[55] The hypothetical **bûm* of [MCT, 161] (cf. [CAD II, 297] and [AHw I, 116]) ascribed to mathematical texts is constructed from *ba-a-šu* and similar forms, which almost certainly correspond to a contracted form *bāššu* (<**bāmat-šu*) of *bāmtum*+possessive suffix *-šu*.

[56] I have noticed only one exception to this rule: the "group 7B" text IM 43993, which uses *letûm*, "to split, to divide, to scatter".

break (into any number of parts)". This thus presents us with a rare case of clear-cut separation of technical and general-language meaning – quite different from what we saw in the case of removal-subtractions.

Kassite survival

We have very little evidence for any kind of mathematics from the Kassite period (c. 1600 to c. 1200), nor indeed indirect evidence of the kind we have for the Kassite unfolding (after late Old Babylonian inception) of systematization of fields like incantation, medicine and extispicy. It seems that the scribal families that took care of the conservation of scribal scholarship did not care for the survival of mathematical sophistication.

One text, AO 17264, looks as an exception to this rule (the dating is made on the basis of palaeography; the dealer claimed the tablet to be from Uruk). It is a procedure text about a very intricate problem, the partition of a trapezoidal field between six "brothers" into strips that are pairwise equal. Actually, the problem is too intricate for its author, and the solution is no mathematical solution. Lis Brack-Bernsen and Olaf Schmidt conclude [1990: 38] after analyzing the text that the problem

> is beyond the capability of Babylonian mathematicians, and it looks as if they have given up in despair in their attempt at solving this problem and just given some meaningless computations that lead to a correct result.[57]

But this is not our primary concern here. More interesting are the problem format and the terminology. The statement first tells the object, and asks an explicit question en.na (much faster to write than en.nam). The prescription starts za.è kì.da.zu.dè, "you, by your proceeding" (in the "southern" spelling of Groups "2" and "3"), and ends *kīam nēpešum* – a formula known only from Old Babylonian texts belonging to Groups "6", "7" and "8". The plane "equalside" is a noun and "comes up" – *ba-se-e-šu šu-li-ma* – the phrase as well as the unorthographic spelling points to "Group 7B". Results are followed by I.DÙ; both Neugebauer and Thureau-Dangin understand this as ^{i}dù~*ibanni*, "it produces" (literally "it builds"), which would be an absolute innovation; the complement *i* suggests that this may indeed

[57] It is indeed not too difficult to construct a statement with adequate parameters and a known solution from the table of squares.

have been the scribe's own understanding. But the spelling igi.dù instead of igi.du$_8$ in IM 55357 suggests that the historical root of the innovation is a reinterpretation of the unorthographic Ešnunna spelling of "seeing" – another (scholar's) folk etymology".

Unorthographic spelling also seems to explain íb.TUG, used twice about the remainder after a removal: As proposed by Thureau-Dangin, the word is likely to stand for *šapiltum*, which would regularly be written íb.tag$_4$.

Accumulation is UL.GAR, as in groups "6A" and "8A", while squaring is UR.KA – apparently a cross-breed between UR.UR (YBC 4662–63, "Group 2") and KA+GAR (TMS XXVI, "Group 8C"). LAGAB, elsewhere used as a logogram for squaring and rectangularization, is used instead to tell the equality of shares (probably intended as si$_8$). "Breaking" is treated as in "Group 7A", mentioning neither that it is "into two" (as in "Group 4") nor the resulting natural half (as habitual elsewhere).

Apart from the spelling of the introductory formula, the features are thus definitely "northern", but vacillating between Groups 6A, 7A+B and 8A+C, with preponderance for the links to "Group 7". If the tablet is really from Uruk, the southern tradition must have been so brutally interrupted that sophisticated mathematics had to be imported anew during the Kassite period. Since dealers are not necessarily to be trusted, the text may also represent the left-overs of the northern tradition without being strictly descended from any of the groups which accident has allowed us to discover.

Fifth-century scholar-scribes

We know that Assurbanipal claimed in the mid-seventh century to be able to perform multiplications (a.rá) and to "detach" reciprocals (*u-pa-ṭar* i.gi),[58] which shows survival of the basic terms of sexagesimal place-value computation within the environment where the future king had received his scribal training. But we have to wait another couple of centuries before two texts containing mathematical problems turn up [Friberg, Hunger & al-Rawi 1990; Friberg 1997]. As can be read in a colophon, these texts belonged to a scholar-scribe from fifth-century (thus

[58] [Ungnad 1917: 41*f*], revised interpretation. Later quotations of the text, such as [Fincke 2003: 111], tend to understand its mathematics less well than Ungnad did.

Achaemenid) Uruk.[59] At least the text carrying a colophon was copied from a wax tablet, probably by the owner.

The problem format in these texts is rudimentary. They start by presenting the situation, probably in grammatically neutral form (sometimes certainly, sometimes the use of logograms *could* hide an intended first person singular), and then mostly specify the question with en (an even more radical abbreviation of en.nam). The prescription is formulated in the second person singular and either devoid of opening formulae or introduced mu nu zu.*ti*, "since you do not know".[60] Sometimes, the prescription is formulated in general terms and not as a specific numerical paradigmatic example. Often, the calculation is made in two ways, "if (*šumma*) 5′ is your cubit" and "if 1 is your cubit", corresponding of the choice of the nindan (12 cubits, *ca* 6 m) respectively the cubit as the basic unit for the sexagesimal calculations. In the Old Babylonian period, the cubit was used as the basic unit for vertical distances only. Could it be that the corresponding metrological table had survived in the scholarly environment but its particular use had been forgotten?

Both texts are concerned with new area metrologies, one based on "broad lines" (cf. note 42 and preceding lines), the other on the standard expectation concerning the grain needed for sowing and for feeding the plough oxen. Both correspond to the habits of genuine surveyors.

Some of the problems are "algebraic" in nature – not derived, however, from the fully developed Old Babylonian discipline but from the simple riddles that had once inspired it.

Part of the terminology for operations has Old Babylonian antecedents. ĝar.ĝar and daḫ, respectively "to accumulate" and "to append", are both used as traditionally (always written logographically). Subtraction, however, is made by

[59] Namely to Samaš-iddina, "son of Nādinu, descendant of Šangi-Ninurta, exorcist from Uruk" [Friberg, Hunger & al-Rawi 1990: 545], dating [Robson 2008: 227–237]. In order to avoid wrong connotations to Catholicism or modern occultism (mathematics is not the only field where wrong connotations turn up!), it might be better to translate the profession of the forefather of the scribal family as "ritual specialist".

[60] nu zu and the syllabic equivalent *lā tid(u-)* also appear in Old Babylonian Groups "1", "3" and "7A", but not as opening formulae for the prescription. Since absence of knowledge is inherent in the problem situation and nu zu its simplest expression, reinvention of the same formula is far from excluded.

"lifting", that is, nim, a term that in Old Babylonian texts had been used occasionally as a logogram for *našûm*, "to raise" – cf. above, after note 10. The transmission in Sumerian must thus have been partially interrupted, and a new translation of Akkadian (or, by now, Aramaic) terms into Sumerian must have taken place. íl, the other logogram for "raising", has conserved its meaning, and the syllabic *našûm* may also be encountered. Constructing a square is *maḫārum* (syllabic, but not *šutamḫurum*). Often, multiplication is a.rá *n* rá, "steps *n* go", similar both to the Ur expression *a* a.rá *b* ù.ub.rá (above, after note 48), and to that of various series texts, a.rá *n* e.tab, "(in) *n* steps repeated". The "equalside" is UR.A, but in order to find numerically the equalside of *A*, the phrase *A*.e àm ti*qe* (ti*qe* = *leqe*, "take") may be used, with the alternative ib.sá, unorthographic for íb.sá = íb. si$_8$. Friberg proposes [Friberg, Hunger & al-Rawi 1990: 509] that the former formulation may be an abbreviated reference to a formula used in a few Old Babylonian tables of inverse squares, *A*.e *s* àm íb. si$_8$.[61]

Results are mostly marked by a preceding enclitic *–ma*, but final results often by igi*mar* or *tammar*, "you see". The general rules may also refer to an intermediate result (which because of the abstract formulation cannot be identified numerically) as *šá ana* igi-*ka* e$_{11}$ *a*, "what for your eye comes up" [Friberg, Hunger & al-Rawi 1990: 536] – a combination of the two ways results were announced in Group "7A", whose closeness to the riddle tradition ("If somebody ...") we noticed.

All in all, these texts, like the Kassite AO 17264, confirm that the "southern" post-Hammurabi traditions as represented by groups "1" through "4" had no conspicuous influence in what little problem culture survived the Old Babylonian collapse. Transmission within scholarly (that is, Sumerian-trained) and less scholarly but still schooled practitioner's environments as well as within orally based milieus of lay practitioners probably participated in the process, but it is difficult to extricate their respective roles.

The Seleucid texts

Three Seleucid problem texts are known: VAT 7848, AO 6484 and BM 34568. A colophon in AO 6484 states that it was written by the astrologer-priest Anu-aba-utēr, member of a scribal family descending from the astrologer-priest Sîn-leqe-unninnī from Uruk. Anu-aba-utēr was active in the early second century [Hunger 1968: 40 #92 and *passim*]. The colophons of the other two texts are destroyed, but

[61] Friberg transcribes à-m as a-an, but that makes no difference.

they appear to come from the same scholar-scribes' environment and to be roughly contemporary.

The problem format is rudimentary. The statement *may* start by stating the object, but mostly only describes the situation, apparently in grammatically neutral form; there is no closing formula, and no explicit question except when it is not clear what is meant. In BM 34568, the prescription starts mu nu zu[ú] (the phonetic complement indicating an Akkadian pronunciation *aššum la tidû*, "since you do not know"), and it can be seen to be meant to be in the second person singular. In the other two texts, there is no opening formula, and the prescription appears to be grammatically neutral.

As concerns the operations, "accumulation" has become ĝar in BM 34568 but remains ĝar.ĝar in the other two. The identity-conserving addition has become *ṭepûm*, mostly in the logographic writing tab – which, we remember, was used for "repetition" in the Old Babylonian texts. Just as in the case of nim in the fifth-century texts, we have evidence of a re-Sumerianization of the vernacular language and thus of interruption of the tradition at the scholarly level.

Similar evidence comes from the terms for subtraction. Beyond nim, which is still used as "lifting up" from the reckoning board, removal may be designated lal, which in Old Babylonian times had been used for comparison "the other way round" (above, after note 44).

Multiplication is *a* GAM *b* or *a* GAM *b* rá, where the easily written repetition sign GAM (in the three-stroke variant) is obviously used as an ideogram corresponding to a.rá but perhaps rather to be understood as "*a* repeated *b* (times)".

All variants of íb. si_8 (the "equalside") have disappeared, and so has the enigmatic fifth-century use of àm in the same function. Instead, these texts ask for the square root of *A* in a purely arithmetical phrase, "how many steps of what shall I go so that *A*?" [62]

Several problem types from the two texts AO 6770 and BM 34568 that have no known antecedents in Mesopotamia turn up in Demotic papyri from the same epoch [Høyrup 2002b]. The scholar-scribes from Uruk never went there, they had nothing to do with the Assyrian, Achaemenid and Macedonian armies and tax collectors that had been customary visitors of Egypt since centuries. Even the contents

[62] *mi-nu-ú* GAM *mi-ni-i lu*-rá *ma lu A*. The genitive *mi-ni-i* removes any possible doubt that GAM really corresponds to a.rá, "steps of", or to "repetitions of".

of the problems thus confirms that the scholar-scribes adopted much of their mathematics from practitioners who did go around the world.

The mathematical terminology of astronomy

Not all of their material, however, came from that source. These ritual specialists and "scribes of (the astrological omen series) *Enûma Anu Enlil*"(or some of them) were also those who produced mathematical astronomy. The tendency toward arithmetization which we see in the transformed question for the square root is likely to have been inspired by their extensive numerical work; even though the many-place tables of reciprocals produced in Seleucid times probably had no direct function in astronomical calculation, even these may be an abstract spin-off from the same numerical practice.

Planetary tables in themselves contain no terminology for the mathematical operations involved in their production. However, another astronomical genre does: the procedure texts.

One of these – BM 42282+42294, a probably Achaemenid text from Babylon or Borsippa – explains the "goal-year method". It contains no problems, so we should not look for any problem format. What we find is a terminology for additive and subtractive operations.

Certain Old Babylonian terms that have disappeared from the Late Babylonian problem texts survive here: *kamārum* (written phonetically) as well as ĝar.ĝar for "accumulation", and zi for subtraction (the latter probably meant as "lifting" since operations on the "hand" are explicitly spoken about; even ĝar.ĝar could be meant as "positing" on the reckoning board, as once in Ur III). But the identity-conserving addition (whether thought of as *waṣābum* or *ṭepûm*) has become tab, as in the Seleucid texts (above, before note 58).[63]

[63] The terminology of a larger number of astronomical procedure texts is described in [Ossendrijver 2012: 19–26], however without chronological distinctions. Since most of the texts are undated, such distinctions are most likely not feasible; in any case, some of the observations will concern fourth of fifth-century texts, many however texts written in Seleucid times or even the first century (still BCE) – but possibly sometimes as copies of earlier texts. For such reasons correlation with the terminologies of Achaemenid and mathematical texts is difficult. Two points may be added to what is said above. Firstly, subtraction is sometimes *šūlûm* (from *elûm*), another word that seems to refer to the taking-up from the reckoning board; secondly, *našûm*, "to raise", may be used about the calculation of a quantity –but in the factitive D-stem *naššûm*, which seems to mean "to make come up", reminding of the Achaemenid expression "what for your eye comes up" which we encountered above.

The "handbook" MUL.APIN [ed. Hunger & Pingree 1989II : 101], known among other places from Assurbanipal's library and not necessarily much older than the initial seventh century, shows us that "raising" (written íl) was still in use, and that the outcome of a calculation might be "seen" (*tammar*). But this was written when mathematical astronomy was at most in its most primitive beginnings. Half a millennium or more separates it from our Seleucid texts.

Texts referred to, with location of publication

AO 6484: MKT I, III.

AO 6770: MKT II.

AO 8862: MKT I.

AO 17264: MKT I.

BM 13901: Thureau-Dangin 1936, MKT III.

BM 15285: MKT I.

BM 34568: MKT III.

BM 42282+42294: Brack-Bernsen & Hunger 2008.

BM 85194: MKT I.

BM 85196: MKT II.

BM 85200 + VAT 6599: MKT I.

BM 85210: MKT I, III

BM 96957: Robson 1996.

CBS 43: Robson 2000.

CBS 11318: Neugebauer & Sachs 1984.

CBS 154+921: Robson 2000.

Db_2 –146: Baqir 1962.

Haddad 104: al-Rawi & Roaf 1984.

IM 43993. Unpublished, courtesy of Jöran Friberg and Farouk al-Rawi.

IM 52301: Baqir 1950b.

IM 52916+52685+52304: Goetze 1951.

IM 55357: Baqir 1950a.

IM 121613. Unpublished, courtesy of Jöran Friberg and Farouk al-Rawi.

IM 52916+52685+52304: Goetze 1951.

Ni 18: Proust 2008.

Plimpton 322: MCT

Str 367: MKT I.

TMS, all texts: TMS.

UET 5,858: Friberg 2000.

UET 5,859: Friberg 2000.

UET 5,864: Friberg 858.

UM 29-15-192: Proust 2008: 180–183.

VAT 7528: MKT I.

VAT 7532: MKT I, III.

VAT 7535: MKT I.

VAT 7848: MCT.

VAT 8521: MKT I.

VAT 8523: MKT I, III.

YBC 4608: MCT.

YBC 4662: MCT.

YBC 4663: MCT.

YBC 4668: MCT.

YBC 4669: MKT I, III.

YBC 4673:MKT I, II, III.

YBC 4675: MCT.

YBC 4698: MKT III.

YBC 4714: MKT I.

YBC 6295: MCT.

YBC 6504: MKT III.

YBC 6967: MCT.

YBC 7289: MCT.

YBC 7290: MCT.

YBC 7302: MCT.

YBC 8600: MCT.

YBC 8633: MCT.

YBC 11126: MCT.

References

[1] AHw: Wolfram von Soden, *Akkadisches Handwörterbuch*. Wiesbaden: Otto Harrassowitz, 1965–1981.

[2] al-Rawi, Farouk N. H., & Michael Roaf, 1984. "Ten Old Babylonian Mathematical Problem Texts from Tell Haddad, Himrin". *Sumer* **43** (1984, printed 1987), 195–218.

[3] Baqir, Taha, 1950a. "An Important Mathematical Problem Text from Tell Harmal". *Sumer* **6**, 39–54.

[4] Baqir, Taha, 1950b. "Another Important Mathematical Text from Tell Harmal". *Sumer* **6**, 130–148.

[5] Baqir, Taha, 1951. "Some More Mathematical Texts from Tell Harmal". *Sumer* **7**, 28–45.

[6] Baqir, Taha, 1962. "Tell Dhiba'i: New Mathematical Texts". *Sumer* **18**, 11–14, pl. 1–3.

[7] Beckman, N., & Kr. Kålund (eds), 1914. *Alfræði íslenzk. Islandsk encycklopædisk Litteratur*. Vol. II. *Rímtol*. København: Samfund til Udgivelse af Gammel Nordisk Litteratur / S. L. Møller, 1914–16.

[8] Blume, F., Karl Lachmann & A. A. F. Rudorff (eds), 1848. *Die Schriften der römischen Feldmesser*. Herausgegeben und erläutert. I. *Texte und Zeichnungen*. Berlin: Weidmann, 1848, 1852.

[9] Brack-Bernsen, Lis, & Olaf Schmidt, 1990. "Bisectable Trapezia in Babylonian Mathematics". *Centaurus* **33**, 1–38.

[10] Brack-Bernsen, Lis, & Hermann Hunger, 2008. "BM 42484+42294 and the Goal-Year method". *SCIAMUS* **9**, 3–23.

[11] Brunke, Hagan, 2011. "Überlegungen zur babylonischen Kreisrechnung". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **101**, 113–126.

[12] Bubnov, Nicolaus (ed.), 1899. Gerberti postea Silvestri II papae *Opera mathematica* (972 – 1003). Berlin: Friedländer.

[13] CAD: *The Assyrian Dictionary of the Oriental Institute of Chicago*. 21 vols. Chicago: The Oriental Institute, 1964–2010.

[14] Castellino, G. R., 1972. *Two Šulgi Hymns (BC)*. (Studi semitici, 42). Roma: Istituto di studi del Vicino Oriente.

[15] Damerow, Peter, & Robert K. Englund, 1987. "Die Zahlzeichensysteme der Archaischen Texte aus Uruk", Kapitel 3 (pp. 117–166) *in* M. W. Green & Hans J. Nissen, *Zeichenliste der Archaischen Texte aus Uruk*, Band II (ATU 2). Berlin: Gebr. Mann.

[16] Durand, Jean-Marie, 1997. *Les documents épistolaires du palais de Mari*. 3 vols. Paris: Éditions du Cerf, 1997–2000.

[17] Eyssenhardt, Franz (ed.), 1858. Macrobius. Leipzig: Teubner.

[18] Fincke, Jeanette C., 2003. "The Babylonian Texts of Nineveh: Report on the British Museum's *Ashurbanipal Library Project*". *Archiv für Orientforschung*, **50** (2003–2004), 111–149.

[19] Foster, Benjamin, & Eleanor Robson, 2004. "A New Look at the Sargonic Mathematical Corpus". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie*, **94**, 1–15.

[20] Friberg, Jöran, 1978. "The Third Millennium Roots of Babylonian Mathematics. I. A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and proto-Elamite Semi-Pictographic Inscriptions". *Department of Mathematics, Chalmers University of Technology and the University of Göteborg*, No. 1978–9.

[21] Friberg, Jöran, Hermann Hunger & Farouk N. H. al-Rawi, 1990. "»Seeds and Reeds«: A Metro-Mathematical Topic Text from Late Babylonian Uruk". *Baghdader Mitteilungen* **21**, 483–557, Tafel 46–48.

[22] Friberg, Jöran, 1997. "'Seed and Reeds Continued'. Another Metro-Mathematical Topic Text from Late Babylonian Uruk". *Baghdader Mitteilungen* **28**, 251–365, pl. 45–46.

[23] Friberg, Jöran, 2000. "Mathematics at Ur in the Old Babylonian Period". *Revue d'Assyriologie et d'Archéologie Orientale* **94**, 97–188.

[24] Friberg, Jöran, 2001. "Bricks and Mud in Metro-Mathematical Cuneiform Texts", pp. 61–154 in Jens Høyrup & Peter Damerow (eds), *Changing Views on Ancient Near Eastern Mathematics*. (Berliner Beiträge zum Vorderen Orient, 19). Berlin: Dietrich Reimer.

[25] Friberg, Jöran, 2007. *A Remarkable Collection of Babylonian Mathematical Texts*. Manuscripts in the Schøyen Collection, Cuneiform Texts I. New York: Springer.

[26] Friberg, Jöran, & Farouk al-Rawi, 2016. *New Mathematical Cuneiform Texts*. Cham etc.: Springer, 2016.

[27] Glassner, Jean-Jacques, 2005. "L'aruspicine paléo-babylonienne et le témoignage des sources de Mari". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **95**, 276–300.

[28] Goetze, Albrecht, 1945. "The Akkadian Dialects of the Old Babylonian Mathematical Texts", pp. 146–151 *in* O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts.* (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society.

[29] Goetze, Albrecht, 1951. "A Mathematical Compendium from Tell Harmal". *Sumer* **7**, 126–155.

[30] Heiberg, J. L. (ed., trans.), 1883. Euclidis *Elementa*. 5 vols. (Euclidis Opera omnia, vol. I-V). Leipzig: Teubner, 1883–1888.

[31] Heiberg, J. L. (ed., trans.), 1912. Heronis *Definitiones* cum variis collectionibus. Heronis quae feruntur *Geometrica*. (Heronis Alexandrini Opera quae supersunt omnia, IV). Leipzig: Teubner.

[32] Heiberg, J. L. (ed., trans.), 1914. Heronis quae feruntur *Stereometrica* et *De mensuris*. (Heronis Alexandrini Opera quae supersunt omnia, V). Leipzig: Teubner.

[33] Høyrup, Jens, 1993. "`Remarkable Numbers' in Old Babylonian Mathematical Texts: A Note on the Psychology of Numbers". *Journal of Near Eastern Studies* **52**, 281–286.

[34] Høyrup, Jens, 1995. "Linee larghe. Un'ambiguità geometrica dimenticata". *Bollettino di Storia delle Scienze Matematiche* **15**, 3–14. English translation http://www.akira.ruc.dk/~jensh/Publications/2010{K}03_Broad lines.PDF.

[35] Høyrup, Jens, 1997. "Hero, Ps.-Hero, and Near Eastern Practical Geometry. An Investigation of *Metrica, Geometrica*, and other Treatises", pp. 67–93 *in* Klaus Döring, Bernhard Herzhoff & Georg Wöhrle (eds), *Antike Naturwissenschaft und ihre Rezeption*, Band 7. Trier: Wissenschaftlicher Verlag Trier, 1997. (For obscure reasons, the publisher has changed □ into ~ and ⊏⊐ into ¤§ on p. 83 after having supplied correct proof sheets).

[36] Høyrup, Jens, 2000. "The Finer Structure of the Old Babylonian Mathematical Corpus. Elements of Classification, with some Results", pp. 117–177 *in* Joachim Marzahn & Hans Neumann (eds), *Assyriologica et Semitica*. Festschrift für Joachim Oelsner anläßlich seines 65. Geburtstages am 18. Februar 1997. (Altes Orient und Altes Testament, 252). Münster: Ugarit Verlag.

[37] Høyrup, Jens, 2002a. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. New York: Springer.

[38] Høyrup, Jens, 2002b. "Seleucid Innovations in the Babylonian `Algebraic' Tradition and Their Kin Abroad", pp. 9–29 *in* Yvonne Dold-Samplonius et al (eds), *From China to Paris: 2000 Years Transmission of Mathematical Ideas*. Stuttgart: Steiner.

[39] Høyrup, Jens, 2002c. "How to Educate a Kapo, or, Reflections on the Absence of a Culture of Mathematical Problems in Ur III", pp. 121–145 *in* John M. Steele & Annette Imhausen (eds), *Under One Sky. Astronomy and Mathematics in the Ancient Near East*. (Alter Orient und Altes Testament, 297). Münster: Ugarit-Verlag.

[40] Høyrup, Jens, 2017. *Algebra in Cuneiform: Introduction to an Old Babylonian Geometrical Technique*. Berlin: Edition Open Access.

[41] Hunger, Hermann, 1968. *Babylonische und assyrische Kolophone*. (Alter Orient und Altes Testament, 2). Kevelaer: Butzon & Kercker / Neukirchen-Vluyn: Neukirchener Verlag.

[42] Hunger, Hermann, & David Pingree, 1989. MUL.APIN: *An Astronomical Compendium in Cuneiform*. (Archiv für Orientforschung, Beiheft 24). Horn, Austria: Ferdinand Berger.

[43] Kahn, Charles H., 2003. *The Verb 'Be' in Ancient Greek*. Indianapolis & Cambridge: Hackett Publishing, [1]1973.

[44] Kramer, Samuel Noah, 1949. "Schooldays: A Sumerian Composition Relating to the Education of a Scribe". *Journal of the American Oriental Society* **69**, 199–215.

[45] Lakatos, Imre, 1976. *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge: Cambridge University Press, 1976.

[46] MCT: O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society, 1945.

[47] MKT: O. Neugebauer, *Mathematische Keilschrift-Texte*. 3 vols. (Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung A: Quellen. 3. Band, erster-dritter Teil). Berlin: Julius Springer, 1935–1937.

[48] Muroi, Kazuo, 2001. "Inheritance Problems in the Susa Mathematical Text No. 26". *Historia Scientiarum*, second series **10**:3, 226–234.

[49] Neugebauer, Otto, 1932. "Studien zur Geschichte der antiken Algebra I". *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*. Abteilung B: *Studien* **2** (1932–33), 1–27.

[50] Neugebauer, Otto, 1934. *Vorlesungen über Geschichte der antiken mathematischen Wissenschaften*. I: *Vorgriechische Mathematik*. Berlin: Julius Springer.

[51] Neugebauer, O., & A. Sachs, 1984. "Mathematical and Metrological Texts". *Journal of Cuneiform Studies* **36**, 243–251.

[52] Oelsner, Joachim, 2001. "HS 201 – eine Reziprokentabelle der Ur III-Zeit", pp. 53–59 *in* Jens Høyrup & Peter Damerow (eds), *Changing Views on Ancient Near Eastern Mathematics*. (Berliner Beiträge zum Vorderen Orient, 19). Berlin: Dietrich Reimer.

[53] Ossendrijver, Mathieu, 2012. *Babylonian Mathematical Astronomy: Procedure Texts*. New York: Springer.

[54] Powell, Marvin A., 1976. "The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics". *Historia Mathematica* 3, 417–439.

[55] Proust, Christine, 2000. "La multiplication babylonienne: la part non écrite du calcul". *Revue d'Histoire des Mathématiques* 6, 293–303.

[56] Proust, Christine, 2005. "À propos d'un prisme du Louvre: Aspects de l'enseignement des mathématiques en Mésopotamie". *SCIAMUS* 6, 3–32.

[57] Proust, Christine, 2008. "Quantifier et calculer: usages des nombres à Nippur". *Revue d'Histoire des Mathématiques* 14, 143–209.

[58] Proust, Christine, 2009. "Deux nouvelles tablettes mathématiques du Louvre: AO 9071 et AO 9072". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* 99, 167–232.

[59] Proust, Christine, 2015. "A Tree-Structured List in a Mathematical Series Text from Mesopotamia", pp. 281-316 *in* K. Chemla & J. Virbel (eds), *Texts, Textured Acts and the History of Science.* Cham : etc., : Springer.

[60] Robson, Eleanor, 1996. "Building with Bricks and Mortar. Quantity Surveying in the Ur III and Old Babylonian Periods", pp. 181–190 *in* Klaas R. Veenhof (ed.), *Houses and Households in Ancient Mesopotamia.* Leiden & Istanbul: Nederlands Historisch-Archaeologisch Instituut te Istanbul.

[61] Robson, Eleanor, 1999. *Mesopotamian Mathematics 2100–1600 BC. Technical Constants in Bureaucracy and Education.* (Oxford Editions of Cuneiform Texts, 14). Oxford: Clarendon Press.

[62] Robson, Eleanor, 2000. "Mathematical Cuneiform Tablets in Philadelphia. Part 1: Problems and Calculations". *SCIAMUS* 1, 11–48.

[63] Robson, Eleanor, 2001. "Neither Sherlock Holmes nor Babylon: a Reassessment of Plimpton 322". *Historia Mathematica* 28, 167–206.

[64] Robson, Eleanor, 2008. *Mathematics in Ancient Iraq: A Social History.* Princeton & Oxford: Princeton University Press.

[65] [Rorizcer, Mathes, 1497], *Geometria deutsch.* [Second edition. Nürnberg: Peter Wagner, c. 1497].

[66] Schöne, Hermann (ed., trans.), 1903. Herons von Alexandria *Vermessungslehre* und *Dioptra.* Griechisch und deutsch. (Heronis Alexandrini Opera quae supersunt omnia, vol. III). Leipzig: Teubner.

[67] Shelby, Lon R. (ed.), 1977. *Gothic Design Techniques. The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer.* Carbondale & Edwardsville: Southern Illinois University Press.

[68] Sjöberg, Åke W., 1976. "The Old Babylonian Eduba", pp. 159–179 *in Sumerological Studies* in Honor of Thorkild Jacobsen on his Seventieth Birthday, June 7, 1974. (The Oriental Institute of the University of Chicago, Assyriological Studies, 20). Chicago & London: University of Chicago Press.

[69] Steinkeller, Piotr, 1979. "Alleged GUR.DA = ugula-géš-da and the Reading of the Sumerian Numeral 60". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* 69, 176–187.

[70] Tanret, Michel, 2010. *The Seal of the Sanga: On the Old Babylonian Sangas of Šamaš of Sippar-Jaḫrūrum and Sippar-Amnānum.* Leiden & Boston: Brill.

[71] Thomsen, Marie-Louise, 1984. *The Sumerian Language. An Introduction to its History and Grammatical Structure.* (Mesopotamia, 10). København: Akademisk Forlag.

[72] Thulin, Carl (ed.), 1913. *Corpus Agrimensorum romanorum.* Vol. I, fasc. I: *Opuscula agrimensorum veterum.* Leipzig: Teubner.

[73] Thureau-Dangin, F., 1936. "L'Équation du deuxième degré dans la mathématique babylonienne d'après une tablette inédite du British Museum". *Revue d'Assyriologie* 33, 27–48

[74] TMB: F. Thureau-Dangin, *Textes mathématiques babyloniens.* (Ex Oriente Lux, Deel 1). Leiden: Brill, 1938.

[75] TMS: Evert M. Bruins & Marguerite Rutten, *Textes mathématiques de Suse.* (Mémoires de la Mission Archéologique en Iran, XXXIV). Paris: Paul Geuthner.

[76] Ungnad, Arthur, 1917. "Lexikalisches". *Zeitschrift für Assyriologie und verwandte Gebiete* 31, 38–57.

[77] Velupillai, K. Vela, 2005. "The Unreasonable *In*effectiveness of Mathematics in Economics". *Cambridge Journal of Economics* 29, 849–872.

[78] Wigner, Eugene, 1960. "The Unreasonable Effectiveness of Mathematics in the Natural Sciences". *Communications in Pure and Applied Mathematics* 13:1, 1–14.

Contact Details:

Jens Høyrup
Section for Philosophy and Science Studies
Roskilde University
Box-260
DK-4000, Roskilde
Denmark

E-mail: `jensh@ruc.dk`